

1N-37  
49840  
P-251

NASA Technical Memorandum 106638

# System Identification of Damped Truss-Like Space Structures

Sasan Armand  
*Lewis Research Center*  
*Cleveland, Ohio*

(NASA-TM-106638) SYSTEM  
IDENTIFICATION OF DAMPED TRUSS-LIKE  
SPACE STRUCTURES Ph.D. Thesis -  
Cleveland State Univ., Mar. 1994  
(NASA. Lewis Research Center)  
281 p

N95-27911

Unclass

May 1995

G3/37 0049840



National Aeronautics and  
Space Administration

# TABLE OF CONTENTS

	Page
<b>ABSTRACT</b>	iii
<b>CHAPTERS</b>	
<b>I. INTRODUCTION</b>	1
<b>II. LITERATURE REVIEW</b>	5
2.1 Introduction	5
2.2 Previous Research	6
2.3 Concluding Remarks	15
<b>III. STRUCTURAL FORMULATION AND METHODOLOGY</b>	17
3.1 Introduction	17
3.2 Equation of Motion with Nonproportional Damping	17
3.3 Solution of the Homogeneous Equation	18
3.4 Formulation for Structural Modification	20
3.4.1 Equations and Derivation for Method 1	21
3.4.2 Equations and Derivation for Method 2	26
3.5 Concluding Remarks	29
<b>IV. RESULTS AND DISCUSSIONS</b>	30
4.1 Introduction	30
4.2 Description of the FORTRAN Program	31
4.3 Description of the Structures Under Study	39
4.4 Weighting Matrices	39
4.5 Four-Degree-of-Freedom System	40
4.5.1 Results for Four-Degree-of-Freedom System	42
4.6 12-BAR Truss	51
4.6.1 Results for 12-BAR Truss, 95 Model, Method 1	72
4.6.2 Results for 12-BAR Truss, 95 Model, Method 2	92
4.6.3 Results for 12-BAR Truss, 90 Model, Method 1	101
4.6.4 Results for 12-BAR Truss, 90 Model, Method 2	108

## **CHAPTERS**

### **IV. RESULTS AND DISCUSSIONS – *continued***

4.7	44-BAR Truss	115
4.7.1	Results for 44-BAR Truss, 90 Model, Method 1	136
4.7.2	Results for 44-BAR Truss, 90 Model, Method 2	150
4.8	Concluding Remarks	164

### **V. CONCLUSIONS AND FUTURE WORK** 167

## **BIBLIOGRAPHY** 169

## **ACKNOWLEDGEMENTS** 176

## **APPENDICES**

A.	FORTRAN PROGRAM	177
B.	SYMBOLS	269
C.	LIST OF TABLES	270
D.	LIST OF FIGURES	273

# **SYSTEM IDENTIFICATION OF DAMPED TRUSS-LIKE SPACE STRUCTURES**

**SASAN ARMAND**

National Aeronautics and Space Administration

Lewis Research Center

Cleveland, Ohio 44135

## **ABSTRACT**

A spacecraft payload flown on a launch vehicle experiences dynamic loads. The dynamic loads are caused by various phenomena ranging from the start-up of the launch vehicle engine to wind gusts. A spacecraft payload should be designed to meet launch vehicle dynamic loads. One of the major steps taken towards determining the dynamic loads is to correlate the finite element model of the spacecraft with the test results of a modal survey test. A test-verified finite element model of the spacecraft should possess the same spatial properties (stiffness, mass, and damping) and modal properties (frequencies and mode shapes) as the test hardware representing the spacecraft. The test-verified and correlated finite element model of the spacecraft is then coupled with the finite element model of the launch vehicle for analyses of loads and stress.

Modal survey testing, verification of a finite element model, and modification of the finite element model to match the modal survey test results can easily be accomplished if the spacecraft structure is simple. However, this is rarely the case. A simple structure here is defined as a structure where the influence of nonlinearity between force and displacement (uncertainty in a test, for example, with errors in input and output), and the influence of damping (structural, coulomb, and viscous) are not pronounced. The objective of this study is to develop system identification and correlation methods with the focus on the structural systems that possess nonproportional damping.

Two approaches to correct the nonproportional damping matrix of a truss structure were studied, and have been implemented on truss-like structures such as the National Aeronautics and Space Administration's space station truss. The results of this study showed nearly 100 percent improvement of the correlated eigensystem over the analytical eigensystem. The first method showed excellent results with up to three modes used in the system identification process. The second method could handle more modes, but required more computer usage time, and the results were less accurate than those of the first method.



# CHAPTER I

## INTRODUCTION

Hooke's law states that in a time-independent study of the stress-strain curve for structures, a linear relationship exists which defines the elastic coefficients between the stress and strain tensors. The material of such structures is linear, elastic, and isotropic. Most static structures that operate within a small range of stress and strain possess such properties, and follow Hooke's law. The time-dependent stress-strain curve for a vibrating structure also possesses a linear relationship, and thus follows Hooke's law; that is, as long as the range of the elastic portion of motion is small when compared to the overall size of the structure in question, the structure is considered linear. This research study will focus only on structures which follow Hooke's law. The Lagrangian coordinate system is used here rather than the Eulerian coordinate system.

The most direct way of deriving the equation of motion is to use D'Alembert's principle, which states that when a structure is placed in a dynamic environment, the resultant forces will be in equilibrium with the inertia forces of the structure. However, the contribution of damping, which is the emphasis of this research study, can not be fully explained when D'Alembert's principle is used. When using D'Alembert's principle, the contribution of damping in the equation of motion appears to be a force that resists the motion. Therefore, the contribution of damping is similar to those of inertia and strain. However, damping offers more than a resisting force. In an effort to better explain the contribution and the role of damping in the equation of motion, Hamilton's principle is used. Hamilton's principle states that the variation of kinetic and potential energy, plus the variation of the work done by the nonconservative forces between any two time instances, must equal zero. Hamilton's principle is meant to be used when the structural system is complex, and problems with establishing vectorial equations may be unavoidable. However, since Hamilton's principle deals with the energy quantities of the structural system and damping forces absorb energy, this principle is used to derive the equation of motion. The variational form of Hamilton's principle can be expressed as (Clough, 1975):

$$\int_{t_1}^{t_2} \delta(T - V)dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0 \quad (1.1)$$

where :

$T$  = kinetic energy of the system

$V$  = potential energy (due to strain and external forces) of the system

$W_{nc}$  = work done by nonconservative forces

$\delta$  = variation taken during two time intervals

The kinetic energy of the system is defined as:

$$T = \frac{1}{2}m\dot{q}^2 \quad (1.2)$$

where :

$m$  = mass of the system

$\dot{q}$  = resultant velocity vector

The potential energy for a structural system with no hidden stresses such as preload, can be expressed as the strain energy,  $U$ , of the system:

$$V = U = \frac{1}{2}kq^2 \quad (1.3)$$

where :

$k$  = stiffness coefficient

$q$  = resultant displacement vector

Since the kinetic energy and the potential energy of the system have been defined, the only way to model and introduce the energy due to damping in equation (1.1) is through nonconservative forces, that is:

$$\delta W_{nc} = F(t)\delta q - c\dot{q}\delta q \quad (1.4)$$

where :

$F(t)$  = externally applied forcing function

$\delta q$  = virtual displacement

$c$  = damping coefficient

According to equations (1.1) and (1.4), if all nonconservative forces equal zero, there is no energy-absorbing media that will zero out the vibration of the structural system. Thus, the role of damping force is not only to resist the motion of the structural system (in accordance with D'Alembert's principle), but also to zero out its vibration (in accordance with Hamilton's principle).

The matrix form of the equation of motion of a structural system can be expressed, using either D'Alembert's principle or Hamilton's principle, as:

$$[M]\ddot{q} - [C]\dot{q} - [K]q = [F] \quad (1.5)$$

In this study, only the homogeneous (steady state) portion of equation (1.5) is considered since the modal properties of the structural system can be determined from

the mass, damping and stiffness matrices. The force vector determines the response of the system when a forcing function is applied to the system. Therefore, equation (1.5) becomes:

$$[M]\ddot{q} - [C]\dot{q} - [K]q = [0] \quad (1.6)$$

In the majority of papers that were reviewed for this research study, the influence of damping was neglected. To leave damping off of the equation of motion is a mathematical convenience, which at times is justified. For instance, if the driving frequency is not in the neighborhood of the natural frequency of the structural system there may not be enough time for the effect of damping to become noticeable with harmonic excitation, when the damping is small and the free vibration is over a short interval of time (Meirovitch, 1967). The flight-like structural systems considered in this research study, however, are excited during a long period of launch (approximately 10 to 20 minutes), and the transient forces applied to the structure are not harmonic in nature. Also, because of rapid changes in the transient forces applied to the flight structure, the steady state portion of the excitation is dominated by the transient portion. Therefore, during such conditions, the influence of damping should not be neglected.

There are several ways to represent damping in the equation of motion. The most important type of damping is viscous damping, where the damping forces are proportional to velocity in magnitude and they act in the opposite direction to the velocity vector. Another important type of damping is structural or hysteretic damping. This type of damping is associated with internal friction dissipation of material, and it is due to cyclic strain. Coulomb damping is another way of including damping in the equation of motion. Coulomb damping is similar to viscous damping. The difference is that the magnitude of damping force is constant over the range of motion and velocity in Coulomb damping (Meirovitch, 1967). In this research study, damping is represented by *linear viscous damping*.

The most popular method used to introduce damping into the equation of motion is attributed to Rayleigh (Weaver, 1987). In this method, the coefficients of the damping matrix are proportional to the stiffness matrix and/or to the mass matrix. In this case, the equation of motion can be decoupled using the same transformation as that for the undamped system. However, in practical structures proportional damping properties do not exist, and the damping coefficients are not proportional to either the mass matrix or the stiffness matrix. In this research study, the damping matrix is *nonproportional damping*.

One of the major sources of damping is the friction experienced by the connections of a structure. Other sources consist of material damping, such as air

resistance. As an example, when considering truss-like structures, a drag force which resists the motion of the structure is produced from the friction of the rubbing action in the connections. This drag force is proportional to the velocity of motion, and thus it can be represented by a viscous damper (Hurty, 1965). The presence of friction has a minor role in the steady state response of the excitation for a periodic forcing function, and thus it can be ignored (Weaver, 1987).

The objectives of this research study are the following:

1) To analytically derive the mathematical equations for system identification and correlation of the nonproportional damping matrix of a finite element model. Two paths (as will be discussed in chapter III) will be taken to identify a structural system.

2) To fine-tune the methods devised in this study to correct the damping matrix of the finite element model of a space truss-like structure. Two classes of space trusses will be studied, namely a plane three-bay truss subsequently referred to throughout this study as a 12-BAR truss, and a three-bay three-dimensional truss subsequently referred to throughout this study as a 44-BAR truss similar to that of Space Station Freedom. MSC/NASTRAN, a general purpose finite element analysis package, will be used to generate the stiffness, mass, and damping matrices. Then, these matrices will be used in the system identification methods of this study. In an effort to produce test-like results, simulation studies of the finite element model of the trusses will be performed. The damping and stiffness coefficients of the two classes of trusses will be varied, and the resulting modal properties, which will represent the test modal properties, will be used in conjunction with the finite element model of the truss to correct the damping and stiffness matrices of the finite element model.

3) To compare the two methods for correction of the damping and stiffness matrices with each other and with the test results for accuracy and robustness. The basis for comparison will be the matrices representing the frequency, the eigenvector, the eigenvelocity, and the damping coefficients.

4) To examine the methods for correction of the damping matrix by using highly damped truss structures. A highly damped truss structure, as used here, is defined as a structure whose real and imaginary portions of the vibration frequency possess the same order of magnitude.

5) To generate the methodology in identification of vibration modes (eigenvectors and eigenvelocities) based on the kinetic energy of the test modes in question.

6) To generate the methodology for combining the test modes which possess the highest kinetic energy in system identification and correction of the damping matrix of the finite element model.

## **CHAPTER II**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

The idea of using modal testing to validate and verify a finite element model has been investigated by other researchers. The results of their work provided a starting point for this present study and will be discussed later in this chapter. Of the many approaches suggested in earlier work, two approaches that considered the influence of damping will be reported. Other innovative approaches that ignored the influence of damping will also be reviewed in this study. The author will also explore their application to model refinement. Overall, the majority of the structural modification methods explored by earlier researchers ignore the influence of damping completely.

In an effort to find out why the influence of damping was not considered in the majority of earlier research, the author decided to contact a researcher who consistently ignored damping. The most reasonable choice for this consultation was Benjamin Wada of the National Aeronautics and Space Administration because he has been involved in spacecraft development, modal survey testing, and system identification. The author spoke to Mr. Wada on the telephone in July of 1992. His opinion was that the measurement of damping coefficients is not practical. Furthermore, when damping is introduced to the equation of motion, the complex modes and mode shapes introduce a larger complexity in structural modification, and thus the influence of damping is neglected. However, the opinions of other researchers have consistently indicated that the measured modes in any structure are complex in nature. The complex modes in a structure verify the existence of the nonproportional damping.

In response to the conversation with Mr. Wada, the author proposes a subsystem-level test for samples of the truss joints. This test can include a stiffness test to measure influence-coefficients in six orthogonal directions, and a modal survey test to identify the spatial properties of the joint. The stiffness test will help in the adjustment and modification of the stiffness matrix of the finite element model of the truss. The modal survey test will aid the measurement of the damping coefficients. If a harmonic forcing function is applied to the joint, damping can be evaluated easily by establishing resonance and adjusting the input frequency such that the response data is  $90^\circ$  out of phase with the applied force (Clough, 1975). When this  $90^\circ$  phase-angle is established, the response caused by the displacement vector and the acceleration vector is zero. Thus, only the velocity vector or damping force will contribute to the motion of the joint. The ratio of maximum force to maximum velocity at this point will represent the damping coefficient. The measured damping coefficients in three orthogonal directions can be used to build the damping matrix of the finite element of the truss.

There are other methods for measuring damping than the one I have proposed. These include the matrix method by Hasselman (Hasselman, 1976), the energy method of Kana (Kana, 1975), and the damping synthesis method suggested by Soni (Soni, 1985), who combines the best features of the matrix method and the energy method. Since the main focus of this study was on system identification and the correlation of damping matrices, no particular effort was devoted to measurement techniques for damping coefficients. The assumption was made that the damping coefficients could be measured by one of these methods.

## 2.2 Previous Research

Although the majority of the research studies in the area of system identification and correlation to date do not consider the influence of damping, some of these methods were explored. These will be discussed here since they form the basis upon which this present research was built.

One of the pioneers in the area of system identification is Baruch (Baruch 1, 1978). The method proposed by Baruch is to minimize the Euclidean distance between the analytical and test stiffness matrices, as follows:

Minimize:

$$g_1 = \| [K]^{-1/2}([K_T] - [K])[K]^{-1/2} \| \quad (2.1)$$

such that:

$$g_2 = (q_T)^T [K_T] (q_T) - [\lambda_T] = 0$$

and

$$g_3 = [K_T] - [K_T]^T = 0$$

The test stiffness matrix can be obtained using the following:

$$[K_T] = [K] + [\Delta K] \quad (2.2)$$

where:

$$\begin{aligned} [\Delta K] = & - [K] (q_T) (q_T)^T [M] - [M] (q_T) (q_T)^T [K] \\ & + [M] (q_T) (q_T)^T [K] (q_T) (q_T)^T [M] \\ & + [M] (q_T) [\lambda_T^2] (q_T)^T [M] \end{aligned}$$

Wei's approach is similar to Baruch's and in fact the set up of all of his equations is the same as Baruch's (Wei, 1980). However, in the process of solving for a test

stiffness matrix, Baruch makes an assumption regarding the symmetricity of the product of the transpose of the Lagrange multipliers matrix, the mass matrix, and the eigenvectors to facilitate the solution. Wie, on the other hand, makes no assumptions, and by adding more algebraic equations, obtains the solution for the test stiffness matrix, as follows:

$$\begin{aligned} [K_T] = [K] - [K](q_T)(q_T)^T[M] - [M](q_T)(q_T)^T[K] \\ + [M](q_T)(q_T)^T[K](q_T)(q_T)^T[M] \\ + [M](q_T)[\lambda^2_T](q_T)^T[M] \end{aligned} \quad (2.3)$$

Another approach similar to Baruch's was proposed by Berman (Berman 2, 1983). In Berman's approach, the mass matrix is modified, as follows:

Minimize:

$$g_1 = \| [M]^{-1/2}([M_T] - [M])[M]^{-1/2} \| \quad (2.4)$$

The test mass matrix can be obtained using the following expression:

$$[M_T] = [M] + [M](q_T)[m]^{-1}([I] - [m])[m]^{-1}(q_T)^T[M] \quad (2.5)$$

where:

$$[m] = (q_T)^T[M](q_T)$$

After updating the mass matrix, Berman takes the same approach as Baruch to update the stiffness matrix. However, Berman uses the mass matrix as the weight matrix. Berman's approach is as follows:

Minimize:

$$g_1 = \| [M]^{-1/2}([K_T] - [K])[M]^{-1/2} \| \quad (2.6)$$

such that:

$$g_2 = (q_T)^T[K_T](q_T) - [p_T]^T = 0$$

$$g_3 = [K_T] - [K_T]^T = 0$$

The test stiffness matrix can be obtained using the following expression:

$$[K_T] = [K] + ([\Delta K] + [\Delta K]^T)$$

where:

$$[\Delta K] = \frac{1}{2}[M](q_T)((q_T)^T[K](q_T) + [\lambda^2_T])(q_T)^T[M] - [K](q_T)(q_T)^T[M] \quad (2.7)$$

One of the deficiencies associated with Berman's approach is that the total mass of the system may change. Therefore, another equation must be included in the set of constraint equations, which forces the summation of the masses of all degrees of freedom to be equal to the total mass. Caesar uses the same approach as Berman, but adds two more constraint equations to the system of constraint equations (Caesar 1, 1983). These additional constraint equations are an equation for the total mass and an equation for the interface loads. Caesar's approach is more complete than Berman's. However, another deficiency with both Berman's and Caesar's methods is that the mass matrix and the stiffness matrix are modified separately. If it is believed that both the mass matrix and the stiffness matrix are inaccurate, then a multi-objective minimization problem may have to be set up to modify all inaccurate matrices at one time because the modal data are functions of the mass matrix and the stiffness matrix together.

Chen uses the theory of matrix perturbation to modify the analytical mass and stiffness matrices (Chen 2, 1983). In Chen's approach, the mass and stiffness matrices are expanded in terms of analytical values plus a modification matrix. The advantage of this approach is the simplicity of it which enables real time operation during the modal survey test. Chen's approach follows.

Using the equation of motion the following orthogonality conditions can be obtained:

$$(q_T)^T [M] (q_T) = [I] \quad (2.8)$$

$$(q_T)^T [K] (q_T) = [\lambda_T^2] \quad (2.9)$$

While defining the following perturbation equations as:

$$[M_T] = [M] + [\Delta M]$$

$$[K_T] = [K] + [\Delta K]$$

$$(q_T) = (q) + (\Delta q)$$

$$[p_T] = [p] + [\Delta p]$$

When substituting the above definitions into orthogonality conditions, and ignoring the second order terms, the result is:

$$[\Delta M] = [M](q)(2[I] - (q)^T [M] (q_T) - (q)^T [M] (q))(q)^T [M] \quad (2.10)$$

$$[\Delta K] = [M](q)(2[\lambda^2] + 2[\lambda][\lambda_T] - (q)^T [K] (q_T) - (q)^T [K] (q))(q)^T [M] \quad (2.11)$$

Sensitivity of eigenvalues and eigenvectors is another approach that has been proposed by several authors to modify the spatial property matrices. Sensitivity analysis begins by expressing the equation of motion:



$$([K] - \lambda_j[M])(q) = [0] \quad (2.12)$$

Let's suppose that there is a design variable, such as the cross section of a beam, and there is no agreement as to how to analytically model this design variable. Thus, the accuracy of this design variable is in question. As a result of this uncertainty, the modal data appear to be different from those of the analytical model. The first step in the sensitivity approach is to determine how sensitive the eigenvalues and eigenvectors are to the design variables. Therefore, the partial derivatives of the eigenvalues and the eigenvectors with respect to the design variable need to be obtained. Next, we use the partial derivatives of the eigenvalues and the eigenvectors as though they were the slopes of a set of linear lines which define the relationship of the test modal data with respect to the design variable. We then use the test modal data to calculate the value of the design variable that satisfies the linear equations. Sutter derives the equation of the partial derivatives of the  $i^{\text{th}}$  eigenvalues and eigenvectors, with respect to the  $j^{\text{th}}$  design variable  $V$ , using equations (2.8) and (2.12), as follows (Sutter, 1988):

$$\frac{\partial \lambda_j}{\partial V_i} = (q_j)^T \frac{\partial [K]}{\partial V_i} (q_j) - \lambda_j (q_j)^T \frac{\partial [M]}{\partial V_i} (q_j) \quad (2.13)$$

$$([K] - \lambda_j[M]) \frac{\partial (q_j)}{\partial V_i} = \frac{\partial \lambda_j}{\partial V_i} [M] (q_j) \frac{\partial [K]}{\partial V_i} (q_j) + \lambda_j \frac{\partial [M]}{\partial V_i} (q_j) \quad (2.14)$$

A problem arises since the equation representing the partial derivative of the eigenvectors, equation (2.14), is singular. Several methods have been introduced in the literature to obtain the solution to the singular equation. Some of these methods are:

1) The Finite Difference Method where the methodology of perturbation is used to determine the old eigenvector using the equation of motion:

$$\frac{\partial q_j}{\partial V_i} = \frac{(q_{j\text{new}}) - (q_{j\text{old}})}{\Delta V_i} \quad (2.15)$$

2) The Modal Method where the partial derivatives of the eigenvectors are expressed as an expansion series:

$$\frac{\partial (q_j)}{\partial V_i} = \sum_{k=1}^N A_{ijk} (q_k) \quad (2.16)$$

where:

$$A_{ijk} = \frac{(q_k)^T \left( \frac{\partial [K]}{\partial V_i} - \lambda_j \frac{\partial [M]}{\partial V_i} \right) (q_j)}{(\lambda_j - \lambda_k)} \quad \text{for } k \neq j$$

For  $k = j$ , equation (2.8) is differentiated to obtain the following expression:

$$2(q_j)^T [M] \frac{\partial(q_j)}{\partial V_i} + (q_j)^T \frac{\partial[M]}{\partial V_i} (q_j) = 0 \quad (2.17)$$

When substituting equation (2.16) into equation (2.17), an expression for the coefficient  $A_{ijk}$  can be obtained, as follows:

$$A_{ijk} = -\frac{1}{2}(q_j)^T \frac{\partial[M]}{\partial V_i} (q_j) \quad \text{for } k = j \quad (2.18)$$

3) The Modified Modal Method where a pseudo static solution of equation (2.14) is obtained. This solution is then used as an initial value to approximate the modal deformation. The mode acceleration method used in transient dynamic analysis is the basis for the modified modal method. The pseudostatic solution is obtained by solving equation (2.14), as follows:

$$\left(\frac{\partial(q_j)}{\partial V_i}\right)_s = [K]^{-1} \left( \frac{\partial \lambda_j}{\partial V_i} [M] - \frac{\partial[K]}{\partial V_i} + \lambda_j \frac{\partial[M]}{\partial V_i} \right) (q_j) \quad (2.19)$$

When solving equation (2.14), the quantity  $\lambda_j [M] \frac{\partial(q_j)}{\partial V_i}$  has been neglected.

When the pseudostatic solution, equation (2.19), is added to equation (2.16), the following is obtained:

$$\frac{\partial(q_j)}{\partial V_i} = \left(\frac{\partial(q_j)}{\partial V_i}\right)_s + \sum_{k=1}^N \bar{A}_{ijk} \quad (2.20)$$

where:

$\bar{A}_{ijk}$  = coefficients of the modified modal method

Equation (2.14) is then substituted into equation (2.20) to obtain expressions for  $\bar{A}_{ijk}$ , as follows:

$$\bar{A}_{ijk} = \frac{\lambda_j(q_k)^T \left( \frac{\partial [K]}{\partial V_i} - \lambda_j \frac{\partial [M]}{\partial V_i} \right) (q_j)}{\lambda_k(\lambda_j - \lambda_k)} \quad \text{for } k \neq j \quad (2.21)$$

$$\bar{A}_{ijk} = -\frac{1}{2}(q_j)^T \frac{\partial [M]}{\partial V_i} (q_j) \quad \text{for } k = j \quad (2.22)$$

4) Nelson's Method where the partial derivative of the eigenvectors is expressed in two solutions, namely, the particular solution  $P$ , and the complementary solution  $C$  (Nelson, 1976). Therefore, the partial derivative of the eigenvectors becomes:

$$\frac{\partial(q_j)}{\partial V_i} = [P] + [C](q_j) \quad (2.23)$$

Nelson then identifies the component of the eigenvector with the largest absolute value and constrains the derivative of that component to zero in order to obtain the complementary solution, as follows:

$$C = - (q_j)^T [M] [P] - \frac{1}{2} (q_j)^T \frac{\partial [M]}{\partial V_i} (q_j) \quad (2.24)$$

Two of the very few authors who considered damping in their formulations for system identification are Ibrahim (Ibrahim 7, 1979) and Carnerio (Carnerio, 1987).

Ibrahim introduced the Time Domain Modal Vibration Technique. In this method, the space-state form of the equation of motion is set up for an  $n$ -degree-of-freedom structure, as follows:

$$[M](\ddot{q}) + [C](\dot{q}) + [K](q) = [F] \quad (1.5)$$

Equation (1.5) can be transformed to the following forms:

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} = \begin{bmatrix} [0] & \vdots & [I] \\ -[M]^{-1}[K] & \vdots & -[M]^{-1}[C] \end{bmatrix} \begin{pmatrix} q \\ \dot{q} \end{pmatrix} + \begin{pmatrix} [0] \\ [M]^{-1} \end{pmatrix} [F] \quad (2.25)$$

or:

$$\dot{X} = AX + BF \quad (2.26)$$

where:

$$X = \begin{pmatrix} (q) \\ (\dot{q}) \end{pmatrix} \quad B = \begin{pmatrix} [0] \\ [M]^{-1} \end{pmatrix} \quad \text{and}$$

$$A = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}$$

The eigenvalues of  $A$  are the roots of the characteristic equation (2.26), thus the structural system can be completely characterized by determining these roots. The  $B$  matrix determines the response of the system when a forcing function is applied to the system. Therefore, in a free vibration environment, the modal and spatial properties of the structural system can be determined only from the  $A$  matrix. Equation (2.26) can be written as:

$$\dot{X} = AX \quad (2.27)$$

When considering a  $n$ -degree-of-freedom structural system, the  $A$  matrix is  $2n$ -by- $2n$ . The upper half of the  $A$  matrix, however, consists of a null matrix and an identity matrix. Therefore, the remaining  $n$ -by- $2n$  ( $2n^2$ ) variables have to be determined to fully identify the structural system.

The  $2n^2$  unknown elements of the  $A$  matrix can be determined by noting that at any instant of time,  $t$ , the relationship between the modal properties in equation (2.27) should hold true. Therefore, equation (2.27) can be written as:

$$\dot{X}_i = AX_i \quad (2.28)$$

If the response data, i.e. displacements, velocities, and accelerations, are known at  $2n^2$  time intervals, equation (2.28) can be solved. In order to determine the response data, Ibrahim proposes measurement of the response data at  $2n^2$  time intervals (Ibrahim 7, 1979). As a result, the upper half of equation (2.28) can be written as:

$$[\dot{X}_1, \dot{X}_2, \dots, \dot{X}_{(2n)}] = A[X_1, X_2, \dots, X_{(2n)}] \quad (2.29)$$

If all the response data from a modal survey test are measured, equation (2.29) can be directly solved for all elements of the lower half of the  $A$  matrix. However, to be able to measure the response data, all modes of interest should be present when the structural system is vibrating freely.

In order to measure the response data so that the modes of interest are present simultaneously, one of the three methods of excitations outlined below may be applied to the structure:

1) An impulse load may be applied. Impulse load possesses a uniform spectral density function over the entire frequency range, and this will provide an ideal scenario to measure all modes. However, the impulse load should have enough energy to excite all the modes necessary to identify the spatial properties. Impulse loads do not possess a large amount of energy. In addition, a true impulse load is unattainable. Therefore, for large structures or highly damped structures, impulse loading may not be a feasible method of exciting all the modes of interest.

2) A burst-random or continuous random input load may be applied. The power spectral density function of such input loading is continuous. This type of loading will excite all modes and provide a scenario where all modes of interests can be measured. However, problems may arise since achieving a true Gaussian random noise is not possible. Thus, prediction of the future values of the response data may not be achieved sufficiently from the measured data.

3) A sine-sweep or sinewave loading may be applied. The sine-sweep should be applied over the frequency range of interest. The time period for this type of loading should be kept small enough so that the possibility of highly damped modes dying out is eliminated. This type of loading can provide a larger amount of energy to the structural system, and may be preferred to impulse loading.

Another method for system identification was proposed by Carnerio (Carnerio, 1987). Carnerio combines sensitivity analysis and component mode synthesis formulation to identify a structural system. In this method, the linearized stiffness and damping coefficients are estimated by minimizing the weighted sum of the squares of the difference between experimental and theoretical modal quantities.

Carneiro sets up the extended, weighted, least-squares objective function in the following fashion:

$$J(p) = (\Delta b)^T [G_b] (\Delta b) + \beta (\Delta p)^T [G_p] (\Delta p) \quad (2.30)$$

where:

$J(p)$  = objective function to be minimized

$(p) = (p_1, p_2, \dots, p_q)$  = truss joint parameters

$$(b) = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ q_1 \\ \vdots \\ q_n \end{pmatrix}$$

$[G_b]$  and  $[G_p]$  are the weighting matrices

$\beta$  = confidence factor between experimental and analytical data

$$(\Delta b) = (b)^x - (b)^c$$

$$(\Delta p) = (p)^a - (p)^c$$

The indexes, x, c, and a, refer to the measured, corrected, and analytical quantities.

The scalar quantity in equation (2.30) is set equal to zero, which means that there is no global relative confidence between the experimental data and the analytical data. Therefore, equation (2.30) can be simplified as:

$$J(p) = (\Delta b)^T [G_b] (\Delta b) \quad (2.31)$$

Equation (2.31) is a standard, weighted, least-squares, objective function, and, after the minimization process, a set of linear equations is obtained, as follows:

$$(\Delta b) = [S] (\Delta p) \quad (2.32)$$

where:

$$[S] = \begin{bmatrix} \partial \lambda_1 / \partial p_1 & \dots & \partial \lambda_1 / \partial p_q \\ \vdots & & \vdots \\ \partial \lambda_{m1} / \partial p_1 & \dots & \partial \lambda_{m1} / \partial p_q \\ \vdots & & \vdots \\ \partial q_1 / \partial p_1 & \dots & \partial q_1 / \partial p_q \\ \vdots & & \vdots \\ \partial q_{m1} / \partial p_1 & \dots & \partial q_{m1} / \partial p_q \end{bmatrix} \quad (2.33)$$

The solution of equation (2.33) can be obtained through iteration; for the  $k^{\text{th}}$  step, the solution will be:

$$(\Delta p^{(k)}) = [S^{(k)}]^+ (\Delta b^{(k)}) \quad (2.34)$$

where:

$[S^{(k)}]^+ =$  Moore–Penrose generalized inverse obtained by singular value decomposition of the matrix in question

The parameters of equation (2.34) are corrected as:

$$(p)^{(k+1)} = (p)^{(k)} + (\Delta b)^{(k)} \quad (2.35)$$

Then, the analytical modal data are recalculated and the procedure is repeated until:

$$\|\Delta p_{(k)}\| \leq \varepsilon \|p_{(k)}\| \quad (2.36)$$

If numerical calculations of the sensitivity matrix are desired by means of a finite difference method, then all the derivatives of the eigenvalues and the eigenvectors need to be determined by solving an eigenvalue problem at each nominal point. This type of process is not only time-consuming, but it also introduces round-off and truncation errors into the future analyses. Carniero uses Nelson's methodology to arrive at the equations for the derivatives of the eigenvalues and the eigenvectors with respect to the joint parameters. The derivative equations can then be used in equation (2.34), and this process eliminates round-off and truncation errors and saves computer usage time.

## 2.3 Concluding Remarks

Ibrahim's methodology is indeed innovative and powerful. One weakness of this method however, is that if the modes of interest are sufficiently far apart, it may be an impossible task to excite the structure so that all modes of interest appear in the response data simultaneously. As a result, the response data required to solve equation (2.29) may be unattainable.

The system identification methodology proposed by Carniero is also powerful. However, two weaknesses can be identified in Carniero's methodology.

1) In an effort to identify the structural system, the analyst should have knowledge of how the structural system behaves under loading, and what the important parameters (such as a beam property, etc.) are, and whether these properties influence the important modes of the structure. Based on this knowledge, the analyst is then able to set up the sensitivity matrix. If the finite element model of the structural system in question is composed of several different properties, then it would be a very tedious and time-consuming task for the analyst to search out the important parameters that influence the important modes.

2) To eliminate the need for learning the dynamics of the structural system through trial and error (e.g. varying joint parameters to determine variations in modes and

mode shape) Carnerio suggests another method. This method deals with determination of the locations of the differences between the finite element model and the experimental results through automatic localization methods such as the methodology developed by Zhang (Zhang, 1987). However, according to Caesar in "Update and Identification of Dynamic Mathematical Models," based on experience, the refinement of a finite element model cannot be handled in an automatic manner, since the convergence behavior is very sensitive. In other words, if an incomplete or irrelevant set of system values is used in finite element model refinement, convergence to wrong values can occur.

As a result of studying the work of other researchers, the author formulated two methods for system identification of structural systems possessing nonproportional damping to be studied. The two methods in this study will not be test-dependent, will not require prior knowledge of the structure under study, and will not use localization techniques which may force the problem to converge to wrong values.



## CHAPTER III

### STRUCTURAL FORMULATION AND METHODOLOGY

#### 3.1 Introduction

This chapter contains the derivation of the equations involved in system identification for a structural system possessing nonproportional damping. Two methods are presented to modify the analytical matrices. Method 1 modifies the analytical damping matrix, and method 2 modifies the analytical damping matrix and the analytical stiffness matrix.

#### 3.2 Equation of Motion with Nonproportional Damping

The linear equation of motion of an  $n$ -degree-of-freedom system in matrix form expressed in generalized coordinates  $q_1, q_2, \dots, q_n$  is:

$$[M](\ddot{q}) + [C](\dot{q}) + [K](q) = [F] \quad (3.1)$$

where:

$(q)$  = the modal eigenvector

$(\dot{q})$  = the modal eigenvelocity

$(\ddot{q})$  = the modal eigenacceleration

For a structural system with nonproportional damping each component of eigenvector  $(q)$  is distinguished not only by amplitude but also by phase; thus, two pieces of information are required to determine each one. It follows that  $2n$  equations are required to determine all components of an  $n$ -degree-of-freedom system in each mode. Therefore, to the  $n$  equation of motion (3.1), must be added another equation giving a system of  $2n$  equations to be solved in the case of nonproportional damping. To overcome this stumbling block the missing  $n$  additional equations are added in the following fashion (Hurty, 1965):

$$[M](\dot{q}) - [M](\dot{q}) = [0] \quad (3.2)$$

Equations (3.1) and (3.2) are combined to give the following matrix equations of the order  $2n$ :

$$\begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix} \begin{pmatrix} (\ddot{q}) \\ (\dot{q}) \end{pmatrix} + \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix} \begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix} = \begin{pmatrix} (0) \\ (F) \end{pmatrix} \quad (3.3)$$

Equation (3.3) can be written as:

$$[A](\dot{y}) + [B](y) = (Y) \quad (3.4)$$

where:

$$[A] = \begin{bmatrix} [0] & \vdots & [M] \\ \hline [M] & \vdots & [C] \end{bmatrix}$$

$$[B] = \begin{bmatrix} -[M] & \vdots & [0] \\ \hline [0] & \vdots & [K] \end{bmatrix}$$

$$(y) = \begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix}$$

$$(Y) = \begin{pmatrix} (0) \\ (F) \end{pmatrix}$$

Matrices  $[A]$  and  $[B]$  are both of the order of  $2n$ , real, and symmetric.

### 3.3 Solution of the Homogeneous Equation

The homogeneous form of equation (3.4) is obtained by setting the equation equal to zero.

$$[A](\dot{y}) - [B](y) = [0] \quad (3.5)$$

In an effort to solve equation (3.5), the  $[B]$  matrix should be checked for singularity. In general, the  $[B]$  matrix will have an inverse when the stiffness matrix  $[K]$  is nonsingular. If the stiffness matrix is singular which will be the case if the system is unconstrained with respect to one or more rigid-body displacements, then such rigid-body modes must be removed from the system. For the test problems in this study there will be no rigid-body modes. The solutions of linear equations such as equation (3.5) are found when the displacements and velocities have the form  $e^{\lambda t}$ .  $\lambda$  is the eigenvalue of equation (3.5).

The orthogonality condition of the normal modes with respect to the stiffness and the mass matrices is the base of this method. For the  $[A]$  matrix, the following orthogonality condition exists.

$$(y)^T[A](y) = [a] \quad (3.6)$$

$$(y)^T[B](y) = [b] \quad (3.7)$$

where:

$[a]$  and  $[b]$  = diagonal complex matrices as a result of orthogonality conditions

note that:

$$(y) = \begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix}$$

$$(\dot{q}) = p(q) \quad \text{and} \quad (p) = \left(\frac{1}{\lambda}\right)$$

Equations (3.6), and (3.7) can be rewritten as:

$$\begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix}^T \begin{bmatrix} [0] & \vdots & [M] \\ \hline [M] & \vdots & [C] \end{bmatrix} \begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix} = [a] \quad (3.8)$$

$$\begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix}^T \begin{bmatrix} -[M] & \vdots & [0] \\ \hline [0] & \vdots & [K] \end{bmatrix} \begin{pmatrix} (\dot{q}) \\ (q) \end{pmatrix} = [b] \quad (3.9)$$

Equations (3.8) and (3.9) can be simplified in the following fashion:

$$(\dot{q})^T[M](q) + (q)^T[M](\dot{q}) + (q)^T[C](q) = [a] \quad (3.10)$$

$$- (\dot{q})^T[M](\dot{q}) + (q)^T[K](q) = [b] \quad (3.11)$$

Another convenient way of expressing equation (3.5) is to invert the  $B$  matrix and premultiply it by the  $A$  matrix, as follows:

$$- [B]^{-1}[A](y) = \lambda(y) \quad (3.12)$$

where:

$$- [B]^{-1}[A] = \begin{bmatrix} [0] & \vdots & [I] \\ \hline -[K]^{-1}[M] & \vdots & -[K]^{-1}[C] \end{bmatrix}$$

In this case the orthogonality condition can be expressed as:

$$(\mathbf{y})^T(-[\mathbf{B}]^{-1}[\mathbf{A}])(\mathbf{y}) = [\mathbf{a}] \quad (3.13)$$

And the following equation can be derived:

$$\frac{1}{p}(\mathbf{q}) + [\mathbf{K}]^{-1}[\mathbf{M}](\dot{\mathbf{q}}) + [\mathbf{K}]^{-1}[\mathbf{C}](\mathbf{q}) = \mathbf{0} \quad (3.14)$$

Equation (3.14) will be the active constraint for method 2.

In a practical structure, which consists of  $n$  degrees of freedom, only a few modes and frequencies ( $m$  modes and frequencies) are measured. The reasons for this are that higher modes require high levels of energy due to their shorter wave length, and that exciting and measuring such modes is not practical. Equations (3.10) and (3.11) become  $m$ -by- $m$  matrices when the practicality of measuring eigenvalues and eigenvectors in a modal survey test are considered. If it was possible to measure all  $n$  modes and frequencies of an  $n$ -degree-of-freedom structure,  $(\mathbf{q})$  would become an  $n$ -by- $n$  matrix, and it would be possible to invert both  $(\mathbf{q})^T$  and  $(\mathbf{q})$  on the left side of equations (3.10), and (3.11) and solve for  $[\mathbf{C}]$  directly. However, since  $(\mathbf{q})$  is an  $n$ -by- $m$  matrix, where  $m$  is less than  $n$ , there exists an infinite number of inverses for  $(\mathbf{q})$ . In an effort to determine the minimum  $[\mathbf{C}]$  which satisfies the requirement of equations (3.10) and (3.11), a minimization problem is set up. The Gauss method is used to minimize the mean square error of  $[\mathbf{C}]$  and  $[\mathbf{C}_T]$  matrices. The subscript  $T$  symbolizes the measured variables. However, in order to make the objective function of the minimization problem more robust the author decided to make the mean square error the weighted function of  $[\mathbf{M}]$  or  $[\mathbf{K}]$ .

### 3.4 Formulation for Structural Modification

Two methods were devised in this study to modify the original finite element model in order to improve the correlation between experimental data and analytical results. Both methods focus on correction of the nonproportional damping. As was discussed previously, due to the limited number of modes and frequencies measured in a modal survey test, and the fact that the eigenvector matrix is not square, the test damping matrix and the test stiffness matrix cannot be obtained from the orthogonality conditions. As a result, a second set of equations is introduced that represents the difference between the test properties and the analytical properties. This second set of equations can be treated like the objective functions in a minimization problem. The

orthogonality conditions, then, can be treated as the constraint of the minimization problem. Therefore, in an effort to obtain a set of test properties, the differences between the test properties and the analytical properties are minimized such that the orthogonality conditions are satisfied at all times.

In method 1, the assumption is made that the analytical mass and stiffness matrices are accurate. Therefore, only the damping matrix will have to be correlated. Method 1 will also be used to identify structural systems with inaccuracies in both the damping and the stiffness matrices. Method 1 will be a single-objective minimization problem with the orthogonality condition as its one constraint.

In method 2, the assumption is made that only the analytical mass matrix is accurate. Therefore, the stiffness and damping matrices will have to be correlated against the test results. Method 2 is a multi-objective minimization problem with one constraint derived from the condensed form of the equation of motion.

### 3.4.1 Equations and Derivation for Method 1

In method 1, it is assumed that the mass and stiffness matrices are accurate, and that only the damping matrix has to be modified. The minimization problem to solve equation (3.10) becomes:

Minimize:

$$g_1 = \frac{1}{2} \| [N]([C_T] - [C])[N] \| \quad (3.15)$$

such that:

$$g_2 = (\dot{q}_T)^T [M] (q_T) + (q_T)^T [M] (\dot{q}_T) + (q_T)^T [C_T] (q_T) - [a_T] = 0 \quad (3.16)$$

where the  $[N]$  matrix is the weighting matrix.

Equation (3.15) is the objective function of the minimization problem, and it represents the difference between the damping matrix from the analytical finite element model and the one from the test results. The subscript,  $T$ , represents the test properties. Equation (3.16) is the constraint equation for the minimization problem and it forces the orthogonality condition as derived in equation (3.10) to be satisfied.

In a structural system the sum of all forces and moments acting on the structure, including all inertia loads resulting from the mass and the mass moment of inertia, should equal zero. This extension of D'Alembert's principle results in symmetric stiffness and damping matrices. Therefore, another constraint which would have been appropriate to add to the above minimization problem was the symmetry condition imposed on the damping matrix, as follows:

$$g_3 = C_T - C_T^T = 0 \quad (3.17)$$

However, the author decided to solve equations (3.15) and (3.16) without considering equation (3.17). If the resulting equation for  $C_T$  is not symmetric, then equation (3.17) will be added to the minimization problem.

In an effort to solve equations (3.15) and (3.16), Lagrange multipliers were used. The advantage of using Lagrange multipliers is that the constraint equation is combined with the objective function to form a single equation, as follows:

$$G = g_1 + 2 \Lambda g_2 \quad (3.18)$$

where:

$G$  = new matrix to be optimized.

$\Lambda$  =  $n$  by  $n$  matrix of Lagrange multipliers.

In order to minimize  $G$ , the first derivative of equation (3.18) with respect to the unknown,  $C_T$ , will be obtained and set equal to zero, as follows:

$$\frac{\partial G}{\partial C_T} = \frac{\partial g_1}{\partial C_T} + 2 \Lambda \frac{\partial g_2}{\partial C_T} = 0 \quad (3.19)$$

The partial derivatives of  $g_1$  and  $g_2$  with respect to  $C_T$  will be evaluated separately and then summed up to fulfill the requirement of equation (3.19).

Using Einstein's rule of summation, equation (3.15) can be rewritten as:

$$g_1 = \frac{1}{2} n_{kl} (c_{T_p} - c_{lp}) n_{pq} n_{kr} (c_{T_n} - c_{rs}) n_{sq} \quad (3.20)$$

where:

$$n = [N]$$

$$c = [C]$$

The partial derivative of equation (3.20) with respect to  $c_{T_{ij}}$  is the following:

$$\frac{\partial g_1}{\partial c_T} = n_{ik}^T n_{kr} (c_{T_{rs}} - c_{rs}) n_{sq} n_{qj}^T \quad (3.21)$$

The matrix form of equation (3.21) is:

$$\frac{\partial g_1}{\partial C_T} = [N]^T [N] ([C_T] - [C]) [N] [N]^T \quad (3.22)$$

As will be discussed in chapter IV, based on the work by Baruch,  $[N]$  was selected to be  $[M](1/2)$  (Baruch 2, 1978). Thus, equation (3.22) can be rewritten as:

$$\frac{\partial g_1}{\partial C_T} = [M]^{-1} ([C_T] - [C]) [M]^{-1} \quad (3.23)$$

The next step is to determine the partial derivative of  $g_2$  with respect to  $C_T$  with:

$$\frac{\partial g_2}{\partial C_T} = \frac{\partial}{\partial C_T} (2 \Lambda(q_T)^T [C_T] (q_T)) \quad (3.24)$$

Again, using Einestien's rule of summation, equation (3.24) can be rewritten as:

$$\frac{\partial g_2}{\partial C_T} = \frac{\partial}{\partial C_T} (2 \Lambda_{kp} q_{T_H}^T c_{T_{iq}} q_{T_\Psi}) \quad (3.25)$$

or:

$$\frac{\partial g_2}{\partial C_T} = (2 \Lambda_{kp} q_{T_H}^T q_{T_\Psi}) \quad (3.26)$$

The terms in equation (3.26) can be rearranged to obtain the following result:

$$\frac{\partial g_2}{\partial C_T} = 2(q_T) [\Lambda]^T (q_T)^T \quad (3.27)$$

Equations (3.23) and (3.27) should be added and set equal to zero to fulfill the optimization requirements set by equation (3.19), as follows:

$$\frac{\partial G}{\partial C_T} = [M]^{-1}([C_T] - [C])[M]^{-1} + 2(q_T)[A]^T(q_T)^T = 0 \quad (3.28)$$

There are two unknowns in equation (3.28), namely  $[C_T]$  and,  $[A]^T$ .

Equation (3.28) provides one optimum point for the difference between the damping matrices. However, the necessary and sufficient condition for a global minimum in equation (3.28) is that the second partial derivative with respect to the test damping matrix be positive definite. This second partial derivative is equal to the mass matrix to the power of  $-2$ , and since the mass matrix is positive definite, the optimum point is a global minimum. The strategy for obtaining the test damping matrix would be first to obtain an expression for the Lagrange multipliers matrix and to use this matrix to solve for the damping matrix.

Solving equation (3.28) for  $[C_T]$  by pre- and post-multiplying each side of the equation by  $[C_T]$  and  $[C_T]$  will result in the following equation:

$$(q_T)^T[C_T](q_T) = (q_T)^T[C](q_T) - 2(q_T)^T[M](q_T)[A]^T(q_T)[M](q_T)^T \quad (3.29)$$

Substituting equation (3.29) into equation (3.10), and solving for  $[A_T]^T$  will result in the following equation:

$$[A_T]^T = \frac{1}{2}((q_T)^T[M](q_T))^{-1} \quad (3.30)$$

$$((\dot{q}_T)^T[M](q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C](q_T) - a_T)$$

$$((q_T)^T[M](q_T))^{-1}$$

Equation (3.30) represents the transpose of the Lagrange multipliers. The substitution of this equation back into equation (3.29) will result in an equation that is a function of the test damping matrix, as follows:



$$[C_T] = [C] - [M](q_T)((q_T)^T[M](q_T))^{-1} \quad (3.31)$$

$$((\dot{q}_T)^T[M](q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C](q_T) - a_T)$$

$$((q_T)^T[M](q_T))^{-1}(q_T)^T[M]$$

The only unknown in equation (3.31) is  $[a_T]$ . This unknown can be obtained using the orthogonality conditions, equations (3.10) and (3.11). Using these orthogonality conditions it can be easily shown that:

$$[b_T] = p_T[a_T] \quad (3.32)$$

Since all the terms in equation (3.11) are given, this equation will be used to determine  $[a_T]$ . Substitution of equation (3.32) into equation (3.11), and solving for  $[a_T]$  results in:

$$[a_T] = \frac{1}{p_T}((q_T)^T[K](q_T) - (\dot{q}_T)^T[M](\dot{q}_T)) \quad (3.33)$$

Substitution of  $[a_T]$  into equation (3.30) results in a single equation for the test damping matrix with all the terms given from the test results and the analytical results, as follows:

$$[C_T] = [C] - [M](q_T)((q_T)^T[M](q_T))^{-1} \quad (3.34)$$

$$((\dot{q}_T)^T[M](q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C](q_T) -$$

$$\frac{1}{p_T}((q_T)^T[K](q_T) - (\dot{q}_T)^T[M](\dot{q}_T)))$$

$$((q_T)^T[M](q_T))^{-1}(q_T)^T[M]$$

We can now conclude by studying equation (3.34) that since all the matrix products are diagonal and symmetric, their algebraic summation will also be symmetric. Therefore, there is no need to add equation (3.17) as a second constraint to the optimization problem.

### 3.4.2 Equations and Derivation for Method 2

In method 2 the basic assumption is that there are inherent inaccuracies in both the stiffness matrix and the damping matrix. That is, there are two variables to be determined, namely, the stiffness and the damping. Therefore, the minimization problem is set up so that the values of both the stiffness coefficients and the damping coefficients are minimized in one united objective function. The constraint for this type of problem should have the influence of all spatial properties. Because of this fact, a condensed form of the equation of motion as derived in equation (3.14), rather than the orthogonality conditions as derived in equations (3.10) and (3.11), will be used for the constraint equation. The advantage of having one constraint equation instead of two is that there is one less Lagrange multipliers matrix to be calculated. The minimization problem is the following:

Minimize:

$$g_1 = \frac{1}{2} \| [N]([C_T] - [C])[N] \| + \frac{1}{2} \| [T]([K_T] - [K])[T] \| \quad (3.35)$$

such that:

$$g_2 = (q_T)^T [M] (\dot{q}_T) + (q_T)^T [K_T] \left( \frac{1}{p_T} \right) (\dot{q}_T) + (q_T)^T [C_T] (q_T) = 0 \quad (3.36)$$

where the  $[N]$  and  $[T]$  matrices are the weighting matrices.

In an effort to solve the aforementioned minimization problems Lagrange multipliers will be used to combine the constrained equation with the objective function. This procedure is similar to the procedure used in method 1. An equation similar to equation (3.18) results from combining equations (3.35) and (3.36), as follows:

$$G = g_1 + 2 \lambda g_2 \quad (3.18)$$

In order to minimize  $G$ , the first partial derivative of equation (3.18) with respect to the unknowns  $K_T$ , and  $C_T$  will be obtained and set equal to zero, as follows:

$$\frac{\partial G}{\partial \mathbf{K}_T} = \frac{\partial g_1}{\partial \mathbf{K}_T} + 2 \lambda \frac{\partial g_2}{\partial \mathbf{K}_T} = \mathbf{0} \quad (3.36)$$

$$\frac{\partial G}{\partial \mathbf{C}_T} = \frac{\partial g_1}{\partial \mathbf{C}_T} + 2 \lambda \frac{\partial g_2}{\partial \mathbf{C}_T} = \mathbf{0} \quad (3.37)$$

Similar to derivation of the partial derivative of  $g_1$  with respect to  $\mathbf{C}_T$  in equation (3.23), the derivation of the partial derivative of  $g_1$  with respect to  $\mathbf{K}_T$  and  $\mathbf{C}_T$  results in:

$$\frac{\partial g_1}{\partial \mathbf{K}_T} = [\mathbf{M}]^{-1}([\mathbf{K}_T] - [\mathbf{K}])[\mathbf{M}]^{-1} \quad (3.38)$$

and

$$\frac{\partial g_1}{\partial \mathbf{C}_T} = [\mathbf{M}]^{-2}([\mathbf{C}_T] - [\mathbf{C}])[\mathbf{M}]^{-2} \quad (3.39)$$

The weighting matrix for the partial derivative of  $g_1$  with respect to  $\mathbf{C}_T$  was chosen to be  $[\mathbf{M}]^{-2}$  which is different from the weighting matrix used in method 1 (as will be explained in chapter IV). The weighting matrix for the partial derivative of  $g_1$  with respect to  $\mathbf{K}_T$  was unchanged.

Derivation of the partial derivative of  $g_2$  with respect to  $\mathbf{K}_T$  and  $\mathbf{C}_T$  (which is similar to the derivation of the partial derivative of  $g_2$  with respect to  $\mathbf{C}_T$  in equation (3.27)), results in:

$$\frac{\partial g_2}{\partial \mathbf{K}_T} = 2\left(\frac{1}{p_T} q_T\right) [\mathbf{A}]^T (q_T)^T \quad (3.40)$$

$$\frac{\partial g_2}{\partial \mathbf{C}_T} = 2(q_T) [\mathbf{A}]^T (q_T)^T \quad (3.41)$$

Substitution of equations (3.38) and (3.40) into equation (3.36) results in:

$$\frac{\partial G}{\partial \mathbf{K}_T} = [\mathbf{M}]^{-1}([\mathbf{K}_T] - [\mathbf{K}])[\mathbf{M}]^{-1} + 2\left(\frac{1}{p_T} q_T\right) [\mathbf{A}]^T (q_T)^T = \mathbf{0} \quad (3.42)$$

Substitution of equations (3.39) and (3.41) into equation (3.37) results in:

$$\frac{\partial G}{\partial \mathbf{C}_T} = [\mathbf{M}]^{-2}([\mathbf{C}_T] - [\mathbf{C}])[\mathbf{M}]^{-2} + 2(q_T) [\mathbf{A}]^T (q_T)^T = \mathbf{0} \quad (3.43)$$

Equations (3.42) and (3.43) provide one optimum point for the differences between the damping matrices and stiffness matrices. However, the necessary and sufficient condition for a global minimum in these equations is that the second partial derivative with respect to the test damping and test stiffness matrices be positive definite. The second partial derivatives are equal to the mass matrix to the power of  $-2$  and  $-4$ , and since the mass matrix is positive definite, the optimum point is a global minimum. Equations (3.42) and (3.43) comprise a system of two equations which is solved simultaneously to obtain expressions for  $K_T$ , and  $C_T$ . However, the test stiffness matrix and the test damping matrix are not the only unknowns in the equations. The Lagrange multipliers matrix is also an unknown matrix which will have to be either eliminated or determined.

Solving equation (3.42) for  $K_T$  will result in the following:

$$[K_T] = [K] - 2[M](\frac{1}{p_T}q_T)[\Lambda]^T(q_T)^T[M] \quad (3.44)$$

Equation (3.14) is used one more time here to eliminate the test stiffness matrix in equation (3.44), and to obtain a solution for the Lagrange multipliers in terms of the test damping matrix.

$$[\Lambda_T]^T = \frac{1}{2}((q_T)^T[M](\frac{1}{p_T}q_T))^{-1} \quad (3.45)$$

$$\begin{aligned} & ((q_T)^T[K](\frac{1}{p_T}q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C_T](q_T)) \\ & ((q_T)^T[M](\frac{1}{p_T}q_T))^{-1} \end{aligned}$$

The next step is to solve equation (3.43) for  $C_T$ .

$$[C_T] = [C] - 2[M]^2(q_T)[\Lambda]^T(q_T)^T[M]^2 \quad (3.46)$$

Substituting equation (3.45) into equation (3.46) results in an equation with only one unknown,  $C_T$ :

$$[C_T] = [C] - [M]^2(q_T)((q_T)^T[M](\frac{1}{p_T}q_T))^{-1} \quad (3.47)$$

$$((q_T)^T[K](\frac{1}{p_T}q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C_T](q_T))$$

$$((q_T)^T[M](\frac{1}{p_T}q_T))^{-1}(q_T)^T[M]^2$$

The test damping matrix appears on both sides of equation (3.47). Therefore, the solution of this equation, which will be explained in chapter IV, is not a straightforward process. However, this equation contains only one unknown,  $C_T$ .

The result of substituting equation (3.45) into equation (3.44) is:

$$[K_T] = [K] - [M](\frac{1}{p_T}q_T)((q_T)^T[M](\frac{1}{p_T}q_T))^{-1} \quad (3.48)$$

$$((q_T)^T[K](\frac{1}{p_T}q_T) + (q_T)^T[M](\dot{q}_T) + (q_T)^T[C_T](q_T))$$

$$((q_T)^T[M](\frac{1}{p_T}q_T))^{-1}(q_T)^T[M]$$

The process for obtaining the test stiffness matrix and the test damping matrix is to first obtain a solution for  $C_T$  from equation (3.47), and then to substitute this matrix into equation (3.48) to obtain a solution for  $K_T$ .

### 3.5 Concluding Remarks

The advantages associated with the two methods suggested in this study are:

- 1) Important modes that will be used for system identification can be measured in various types of modal survey tests, and not all modes have to appear in a single test run, which the Ibrahim methodology requires (Ibrahim 7, 1979).
- 2) Prior knowledge of the structural system, which the Carnerio methodology requires (Carnerio, 1987), will not be needed to identify the structural system.
- 3) Localization techniques will not be needed in order to identify the structural system, which the Carnerio methodology also requires.
- 4) The mass matrix, which is usually the most accurate matrix of the three spatial property matrices, does not have to be modified as is done in Berman's method (Berman 2, 1983).

## **CHAPTER IV**

### **RESULTS AND DISCUSSIONS**

#### **4.1 Introduction**

In this chapter an overview of the methods employed to solve the equations derived in chapter III via a FORTRAN program is given. Also, two classes of structures will be modeled and the results will be fully discussed.

The structures to be studied in this chapter are a 3-bay plane truss and a 3-bay space truss. Both trusses are supported at one end, which means that they are cantilevered. As such they do not possess rigid body motion. The remaining degrees of freedom (DOF) of the plane truss can only translate in two translational orthogonal directions in the plane of the truss. The remaining DOF of the space truss can only translate in three orthogonal directions, and the rotational DOF are constrained.

Since in this study no actual structure with measured modal data is used, the FORTRAN program has been designed to perform system identification and to provide simulated modal data. Therefore, we will use the modal data from the simulation portion of the FORTRAN program rather than sets of modal data from an actual modal survey test.

Although the formulations of method 1 of this study modify only the damping matrix, this method will also be applied to the structures whose stiffness matrix contains inherent inaccuracies. In fact, the same structures will be used for both methods. The same structures will be used in order to examine method 1 for structures with inaccuracies in both the damping matrix and the stiffness matrix, and to be able to compare the results of method 1 with those of method 2. It would be an impossible task to compare the effectiveness of methods 1 and 2 without comparing their respective numerical results.

In order to examine the effectiveness of each method and each computer run, two basic sets of data will be provided and tabulated, namely the eigenvalues and the eigenvelocities. The reason for providing the eigenvelocities as opposed to eigenvectors is that eigenvelocities provide information on both the eigenvalues and the eigenvectors at the same time because the eigenvelocities are the product of the eigenvalues and the eigenvectors. Therefore, the method selected for this study and the FORTRAN program to measure the closeness between the eigenvelocities of the test model and those of the analytical model is the most stringent criterion possible.

The numerical results of the system identification and correlation for the 3-bay plane and space trusses will be presented through figures and tables. A discussion of the

results, mainly the eigenvalues and eigenvelocities resulting from the modified properties, will be presented for each truss.

## 4.2 Description of the FORTRAN Program

A 5,400-line FORTRAN program was written for this study to provide the simulated test modal data, and to perform system identification and correlation. The subroutines for complex eigenvalue and eigenvector extractions were obtained from LAPACK and linked to the main FORTRAN program. LAPACK is a national library of mathematical subroutines. The first portion of this program contains PARAMETER cards, and flags cards that control the functions of the program. The second portion of the program reads and writes the modal data and spatial data. The  $A$  and  $B$  matrices are formed by the third portion of the program. The fourth portion of the program contains the subroutine calls for eigenvalue and eigenvector extraction. The solutions to the equations for various methods explored in chapter III are contained in the remainder of the program.

The FORTRAN program contains two options for reading the analytical spatial properties of a structural system. It can read the elements of the mass matrix, the stiffness matrix, and the damping matrix row by row. Alternatively, it can read the data from an OUTPUT4 file format generated by an MSC/NASTRAN finite element program. The OUTPUT4 file is generated by including the DATA MATRIX ABSTRACTION (DMAP) in a MSC/NASTRAN input file. Both options for reading data can be activated via flag IREAD.

As previously mentioned, a simulation program was included as part of this FORTRAN program, in which the test modal properties are generated and saved for the system identification portion of the program. The simulation portion is activated through the IINPUT flag. If the simulation portion of the program is not activated, then the modal properties of the test model will be read for system identification and correlation activities.

Three options for eigenvalue and eigenvector extraction are available in this FORTRAN program. For the structural models that do not possess rigid-body motion, the CG subroutine is used. Prior to using the CG subroutine, the  $B$  matrix is inverted and premultiplied by the  $A$  matrix. This matrix product is then used together with the CG subroutine to solve for the complex eigenvalues and eigenvectors. However, if the structure contains rigid-body motion, then the  $B$  matrix is not invertable, since the stiffness matrix is singular. Two subroutines have been provided for such occasions. The LZHES and LZIT subroutines by Kaufman closely resemble the QZ method, in which the  $A$  matrix is reduced to an upper Hessenberg form, and the  $B$  matrix is reduced to an upper triangular form (Kaufman 1, 1974). The advantage of using LZHES and

LZIT is that elementary transformations are used to reduce the  $A$  matrix and the  $B$  matrix as opposed to orthogonal transformation. This choice of transformations is more efficient when either the  $A$  matrix or the  $B$  matrix is complex. However, for large problems, the LZHES and LZIT subroutines are not stable. For structures possessing rigid body motion, a second set of subroutines is provided, CQZHES, CQZVAL, and CQZVEC. These subroutines are based on the QZ method where the  $A$  matrix and the  $B$  matrix are reduced to an upper Hessenberg matrix and the upper triangular matrix using orthogonal transformations. It is best to use these three subroutines for larger structures. For the classes of structures in this study, the CG subroutine was used since the transformations in the other subroutines were too time-consuming, and the eigenvectors were not as accurate.

The next step in the FORTRAN program is to store the modal properties on tape if the simulation portion of the program is activated, or to continue with correlation activity. When comparing the measured modal properties, specifically the eigenvectors and eigenvelocities from a modal survey test with those from a finite element model, there should be common ground for this comparison. In this FORTRAN program the maximum value of the real part of the eigenvelocities is the common ground.

Prior to system identification activity, the eigenvelocities and eigenvectors were normalized based on the total kinetic energy of the structural system. The idea of this normalization is that as each complex mode is excited, the kinetic energy of the system will vary. The amount of kinetic energy of the lower modes is usually higher, since a larger portion of the structure, or more mass, is in motion. The normalization process was carried out by determining the kinetic energy associated with each mode while using the following equation:

$$T_n = \frac{1}{2} (\dot{q}_n)^T [M] (\dot{q}_n) \quad (4.1)$$

where:

$T_n$  = kinetic energy associated with mode  $n$

The kinetic energy values of each mode are then summed up and the net value represents the total kinetic energy of the structural system. The total kinetic energy of the system is then set equal to 100 percent, and the individual contribution (kinetic energy) of each mode is obtained based on the total energy. This process is performed by dividing the kinetic energy of each mode by the total kinetic energy times 100 percent. The kinetic energy values associated with a structure are output so that the analysts can visually examine these values and determine which modes are important for correlation activity. Obviously, modes possessing lower kinetic energy values may not be as important as the modes possessing higher kinetic energy values. The reason for this is



that modes that possess lower kinetic energy values have short wavelengths and are more difficult to produce in a modal survey test. In addition, the modes possessing low kinetic energy are not usually load-producing modes at supporting points of the structure. It should be emphasized here that the benefit of having well-correlated stiffness and damping matrices is so that they can be used later to generate forces and moments and compare these values with the design loads. Therefore, the modes that do not contribute or have a very small contribution based on their kinetic energy values can be ignored.

Table I and figure 1 represent the kinetic energy values associated with modes one through five of the 12-BAR truss. As can be observed, the kinetic energy values for a structural system with nonproportional damping are complex. Therefore, both real and imaginary parts of the kinetic energy values have been shown. In figure 1, the kinetic energy associated with modes two through four are the highest, and for higher modes, this value diminishes. Also it should be noted that modes two through four possess approximately the same amount of kinetic energy. The reason for this, which will be explored in the modal deformation plots later in this paper, is that the shapes of these modes and the respective eigenvectors are close. We can conclude that it is important to include one of the modes between one and four in the set of modes for correlation activity.

Next, several solutions were included in the FORTRAN program for the purposes of modifying one or two spatial property matrices of the structural system, and to determine the new or correlated eigensystem. They are the Direct Correction, the Stiffness Correction, the Damping Correction, and the Damping-Stiffness Correction. The Direct Correction method is used when the number of modes is the same as the number of DOF. In the Direct Correction method, equation (3.10) is employed to determine the test matrix,  $C_T$ . It should be noted that in order for the Direct Correction method to work, the eigenvector matrix should be square or it cannot be inverted to proceed with the solution.

The method of Stiffness Correction was tailored based on Sidhu's methodology (Sidhu, 1984), as follows:

$$[E] = [K]([K]^{-1} - [K^*_{T}]^{-1})[K] \quad (4.2)$$

where:

$[E]$  = Error matrix, error between analytical and test stiffness matrix

$[K^*_{T}]$  = Pseudo test stiffness matrix

The pseudo test stiffness matrix is obtained by solving equation (3.11) as if the system of equations is underdetermined, as follows:

$$[K^*] = (q^T)([b_T] + (\dot{q}_T)^T[M](\dot{q}_T))(q_T)^T \quad (4.3)$$

The success of the Stiffness Correction method was extremely limited for structural systems with nonproportional damping matrices, and at times the results obtained appeared absurd. The reason for this is that the pseudo test stiffness matrix, as expressed in equation (4.3), is one of the infinite solutions, and this particular solution may be mathematically correct, but not physically correct. Therefore, no results using Sidhu's methodology have been included here. The theory behind including Sidhu's method was to first locate errors in the damping and/or stiffness matrices so that they could be corrected.

The Damping Correction method was tailored by using method 1 discussed in chapter III and equation (3.34). Three attempts are made in this correction method to obtain the test spatial properties. In the first attempt, the test damping matrix that uses equation (3.34) is calculated and used to obtain the modal properties. In the second attempt, the imaginary portion of the test damping matrix using equation (3.34) is set equal to zero, and the modal properties are recalculated. In the third attempt, the weighting matrix is changed to a stiffness matrix, and the problem is repeated.

The Stiffness–Damping Correction uses method 2, discussed in chapter III, and equations (3.47) and (3.48). As can be observed in equation (3.47), the unknown,  $C_T$ , appears on both sides of the equation. Several options for solving the test damping matrix and the test stiffness matrix were followed:

Option 1: The damping matrix is determined with equation (3.47) through an iterative process. Since the test damping matrix is unknown, the analytical damping matrix is assumed for the initial value. This first assumed value is then substituted into the right side of equation (3.47), and a test damping matrix is calculated. The new test damping matrix is substituted into the right side of equation (3.47), and a second test damping matrix is calculated. This process is repeated until some arbitrary convergence criterion is met. When the convergence criterion, based upon the test damping matrix, is met, then equation (3.48) is employed to determine the test stiffness matrix. The test modal properties are calculated by using the test damping matrix and the test stiffness matrix. The success of this option depends upon how close the test damping matrix and analytical damping matrices are. Several attempts were made to use different weighting matrices in equation (3.47), but these attempts did not change the success

of this iterative process, which depends upon the closeness of the test damping matrix and the analytical damping matrix. For large problems, the iterative process as described here totally failed. For small structural systems with small differences between the test and analytical damping matrices, such as a four DOF structural system, this iterative process succeeded.

Option 2: The damping matrix is determined by using equation (3.47) through a random search. This random search consists of assuming values for the elements of the analytical damping matrix, and solving equation (3.48) with these values. It should be noted that the assumed values for the test damping matrix were within  $\pm 10$  percent of the analytical matrix. The success of the random search depends upon the assumed values, and at times even for a small structural system, such as a four DOF structure, the process does not converge.

Option 3: The damping matrix is determined by using equation (3.47) through a systematic search. The systematic search consists of setting the test matrix equal to the analytical matrix and solving for the test stiffness matrix using equation (3.48) which also solves the eigensystem. If the new eigenvalues and the orthogonality matrices as expressed in equations (3.10) and (3.11) are within some arbitrary tolerance of the measured eigenvalues and the measured orthogonality matrices, then a solution is obtained. If not, the elements of the damping matrix are increased or decreased in a systematic fashion (through DO loops) and the above process is repeated until a solution is obtained. The increments or decrements are usually a small percentage of each element of the damping matrix, approximately 10 percent. The systematic search is extremely successful, and in most cases after two to five iterations, an acceptable solution is obtained.

Of the three options listed above, option three was chosen for this study due to its repeated success. However, in the process of working with different structures there were some isolated cases where no convergence was attained. Because the solution to option 3 is through an iterative process, at times there are no solutions.

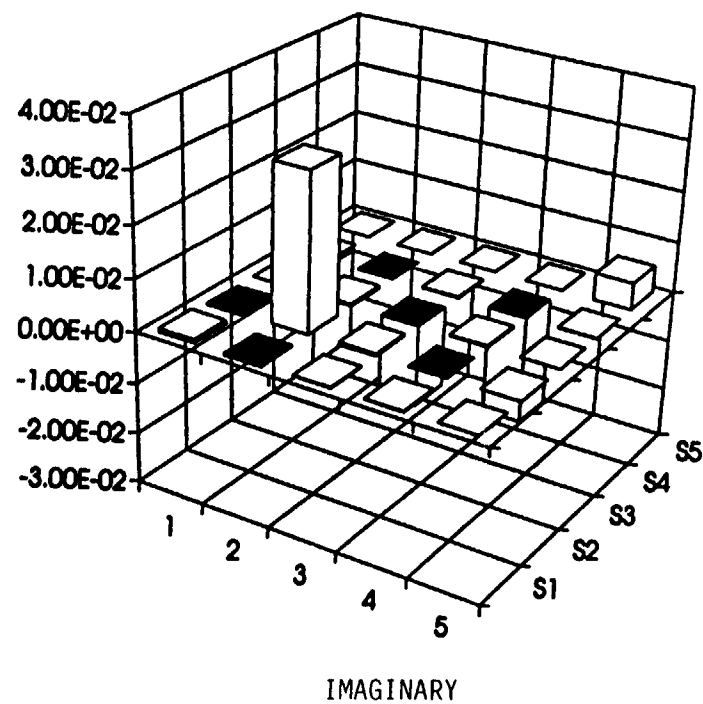
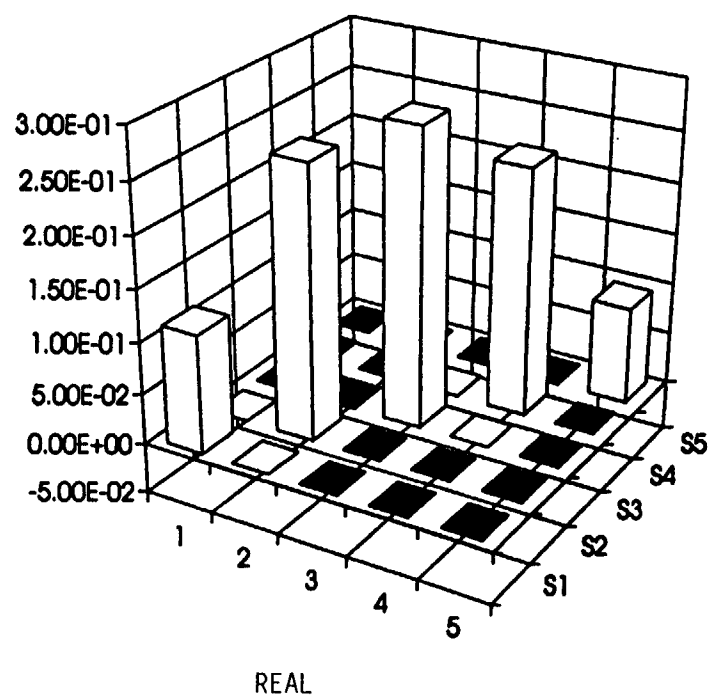
Since the success of the solution to method 2 of this study is through an iterative process, convergence criteria had to be devised. The convergence criteria are determined in two steps. Step 1 compares the calculated eigenvalues with those of the test eigenvalues. If the eigenvalues are within  $\pm 0.1$  Hz, then the FORTRAN program proceeds with step 2. If not, the FORTRAN program varies the coefficients of the damping matrix and solves for another set of eigenvalues until the  $\pm 0.1$  Hz tolerance criterion is met. Step 2 compares the calculated kinetic energy values with those of the analytical model. The basis of comparison is less than a 1 percent difference between the respective kinetic energy values. If the criterion in step 2 is met, then the program outputs the solution. The reason for using the kinetic energy values rather than the

eigenvelocities is that the size of the kinetic energy matrix ( $m$ -by- $m$ ) is smaller than the size of the eigenvelocity matrix. Also, the kinetic energy matrix is a diagonal matrix which contains  $m$  terms, but the eigenvelocity matrix has a larger set of terms,  $m$ -by- $n$  terms to be exact.

Other aspects of the FORTRAN program, for example the subroutines for matrix operations, matrix inversion, output, and eigensystem solutions, will not be discussed here. They are standard subroutines which are used in most program codes.

**TABLE I**  
**KINETIC ENERGY OF THE 12-BAR TRUSS**

	Real					Imaginary				
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 1	Column 2	Column 3	Column 4	Column 5
Row 1	1.15E-01	2.19E-05	-4.31E-06	2.03E-06	-1.19E-06	7.12E-04	-3.03E-04	8.47E-05	2.59E-05	1.62E-05
Row 2	2.19E-05	2.64E-01	-1.33E-05	1.03E-06	-2.47E-06	-3.03E-04	3.07E-02	4.04E-04	-1.39E-05	1.72E-05
Row 3	-4.31E-06	-1.33E-05	2.88E-01	1.42E-05	-5.67E-07	8.47E-05	4.04E-04	-1.57E-02	2.03E-04	1.71E-05
Row 4	2.03E-06	1.03E-06	1.42E-05	2.40E-01	1.54E-06	2.59E-05	-1.39E-05	2.03E-04	-2.10E-02	6.17E-05
Row 5	-1.19E-06	-2.47E-06	-5.67E-07	1.54E-06	9.36E-02	1.62E-05	1.72E-05	1.71E-05	6.17E-05	5.27E-03



**Figure 1: Kinetic Energy Values of 12-BAR Truss, 5 Mode Correlation**  
**Energy of Mode 1 Is S1-1, Energy of Mode N Is SN-N**

### 4.3 Description of the Structures Under Study

For the purpose of system identification, two classes of structures were considered and modeled in this study, a plane three-bay truss which is referred to as the 12-BAR truss, and a three-bay space truss which is referred to as the 44-BAR truss. The reason for modeling two trusses was to show the success of the methodology and formulations presented in chapter III for a variety of trusses. The intent here was to show success for a class of structures, such as the 12-BAR truss, where the bending vibration modes are dominant over the axial vibration modes, with no torsional vibration modes present. Also, the author wanted to examine the methodology on a class of structures, such as the 44-BAR truss, where all vibration modes exist at the same time, and they may couple during excitation. The bending and torsional vibration modes of the 44-BAR truss can couple depending upon the combination of elastic and geometric properties of the truss. The lengths of the members of both trusses were varied between 10 and 100 inches. The success of the results was invariant of the range of dimensions used.

Nearly the entire methodology (methods 1 and 2) in this study was developed on a simple four-degree-of-freedom system first. The reason for the development of the methodology on a four-degree-of-freedom system is that its spatial property matrices are small enough to check the operations performed on them with another mathematical software. The software chosen for checking the FORTRAN program is MATHCAD, which is a commercially available software.

The following terms, used in the tables and figures of chapter IV, are defined as follows:

**Analysis matrices or analytical matrices:** the matrices from the mathematical model or the finite element model which have been generated based on the dimensions specified in the drawings of a structure.

**Test modal properties:** the measured modal properties from a modal survey test.

**Correlated modal properties:** the modal properties obtained by using the test matrices. The test matrices are obtained by a system identification process which is outlined in method 1 and method 2 of this study.

### 4.4 Weighting Matrices

One of the most difficult tasks in this study was to determine the weighting matrices described in equations (3.15) and (3.35). This effort involved experimenting with various spatial property matrices, their matrix products, or their ratios in the

equations of method 1 and method 2, and observing the results. The intent was to make the differences, as described in equations (3.34), (3.47), and (3.48), in the damping matrix and/or the stiffness matrix as small as possible. The difficulty associated with determining the weighting matrices is the fact that the elements of some of the matrix products are large, and at the same time the elements of other matrix products in the same equation are too small for further matrix operations such as addition or subtraction.

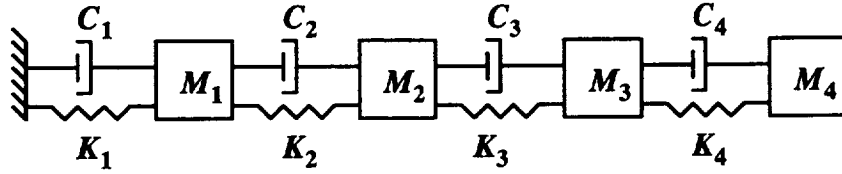
When exercising method 1, by using various weighting matrices, the author's observation of the behavior (success or failure) of equation (3.34) led him to the following conclusion: as long as the elements of the difference matrix product (that are being subtracted from the analytical damping matrix) are approximately 10 to 100 times smaller than the elements of the analytical damping matrix, the system identification is successful. The weighting matrix,  $N$ , used in equation (3.15) and in the FORTRAN program, is the mass matrix to the power of  $-1/2$ , and the result is quite successful. Depending on the magnitude of the elements of the stiffness matrix when they are compared to the elements of the mass matrix to the power of  $-1/2$ , the stiffness matrix may also be used as the weighting matrix. For the two classes of structures in this study, the stiffness matrix was not successful. However, as part of the FORTRAN program for method 1, the inverse of the stiffness matrix (to be used as the weighting matrix for equation (3.34)) was left for future use. The motivation for leaving the inverse of the stiffness matrix in equation (3.34) and the FORTRAN program was that there may be some future structures that can successfully work with this inverse. The system identification methodology proposed and tried by some authors such as Berman or Baruch also use the mass matrix to the power of  $-1/2$  as the weighting matrix. However, their methodology is for structures possessing only normal modes (zero damping).

In an effort to determine the weighting matrix for method 2, the same procedure as described above was used. The trial and error used for method 2, however, led the author to choose the inverse of the mass matrix as the weighing matrix for  $N$  and the mass matrix to the power of  $-1/2$  for  $T$  in equation (3.35). With the combination of the weighting matrices as described here, when an iterative procedure (refer to section 4.2) is used to obtain the test damping matrix in equation (3.47), the process is successful.

#### **4.5 Four-Degree-of-Freedom System**

The four-degree-of-freedom system is shown in figure 2.





**Figure 2: Four-Degree-Of-Freedom System**

We can see in figure 2 that the system is supported at one end. Thus the system does not possess rigid body motion. The following spatial properties were assumed for the analytical model, and these properties were input and read by the FORTRAN program :

$$[M] = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \quad [C] = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad [K] = 1000 \begin{bmatrix} 25 & -10 & 0 & 0 \\ -10 & 15 & -5 & 0 \\ 0 & -5 & 10 & -5 \\ 0 & 0 & -5 & 5 \end{bmatrix}$$

Units :

$$M = \frac{lb - sec^2}{in} \quad C = \frac{lb - sec}{in} \quad K = \frac{lb}{in}$$

Since the purpose of the four-degree-of-freedom system was to mathematically check the methodology and the computer programming, only one of the spatial property matrices was varied. The varied spatial property matrix was the damping matrix. However, the author decided to vary all of the damping coefficients in order to exercise all of the vector columns in the formation of equations (3.34), (3.47) and (3.48). In this way, if any errors occurred in the program, they would show up in the results of the eigenvalues and eigenvectors. Also, in an effort to determine the robustness of the methodology and the formulations of chapter III, the author decided to vary the damping coefficients by 10 percent of the analytical values. In engineering applications, as a rule of thumb, variations of 10 percent and above are considered to be extreme. Thus, the measured damping coefficients should not be over the analytical values by 10 percent. The resulting test damping matrix for the simulation computer run is the following:

$$[C_T] = \begin{bmatrix} 1.8 & -0.9 & 0 & 0 \\ -0.9 & 1.8 & -0.9 & 0 \\ 0 & -0.9 & 1.8 & 0 \\ 0 & 0 & -0.9 & 0.9 \end{bmatrix}$$

Units :

$$C = \frac{lb - sec}{in}$$

#### 4.5.1 Results for Four-Degree-of-Freedom System

After the simulation computer runs were completed, the test modal data were saved on tape for system identification purposes. Next, the system identification was begun by exercising method 1 and method 2, presented in chapter III. For the purpose of system identification in method 1, one mode (mode 1), two modes (modes 1 and 2) and three modes (modes 1, 2, and 3) were selected and used. The discussion of some of the results follows:

**1) Symmetry of Test Matrices:** To determine if the resulting correlated spatial properties are symmetric, the test stiffness and test damping matrices were printed out for one-mode and two-mode system identification. These coefficients are tabulated in table II, and we can see that the matrices are symmetric. Therefore, as was discussed in section 3.4.1, there is no need to add another constraint as expressed in equation (3.17) in order to force the test spatial matrices to be symmetric.

**2) Method 1, One-Mode System Identification:** The results of this computer run are tabulated in table III. Table III shows that the improvement of the first frequency is nearly 100 percent. Also, the eigenvelocities in table IV show the same improvement, that is, nearly 100 percent recovery from the analytical results. It should be noted that sometimes during a system identification, the modes and their respective conjugates switch places. Because of this potential switching, the imaginary portion of the correlated eigenvelocities have different signs from the ones of the test eigenvelocities. Overall, the results of one-mode system identification appear to be quite satisfactory.

**3) Method 1, Two-Mode System Identification:** The results of this computer run are tabulated in tables V and VI. Table V presents the improvement of the first frequency as nearly 100 percent, and the improvement of the second frequency as 100 percent. In addition, the eigenvelocities in table VI show the same improvement, that is, nearly 100 percent recovery from the analytical results for the

first mode and 100 percent recovery for the second mode. Overall, the results of the two-mode system identification appear to be quite satisfactory.

**4) Method 1, Three-Mode System Identification:** The results of this computer run are tabulated in tables VII and VIII. In table VII we can see that the improvement of the first frequency is nearly 100 percent, and the improvement of the second and third eigenvalues have diverged to incorrect values. In table VIII we can see the same improvement and divergence in the eigenvelocities, that is, nearly 100 percent recovery from the analytical results for the first mode and incorrect values for the second and the third modes. Overall, the results of the three-mode system identification are unsatisfactory. There are several reasons for the divergence of the second and the third modes, the second and third modes are local modes, while the first mode is a system mode. This means that because of the large differences between the analytical damping matrix and the test matrix the local modes are too far off to correlate them. Another reason for the divergence is that in a practical structure usually only a small percentage of the spatial properties of the test hardware are different from those of the analytical model. In the four-degree-of-freedom system 100 percent of the damping coefficients were varied, but in reality only a small percentage of the damping coefficients should be different.

**5) Method 2, Two-Mode and Three-Mode System Identification:** The results of these computer runs were not tabulated because no improvements occurred in the analytical eigenvalues and eigenvectors. The cause of the convergence to incorrect eigenvalues and eigenvectors is the large changes made in the damping matrix. Also, as noted in section 4.1, a solution which depends on an iterative process does not always converge to correct values.

It should be noted again that the only reason for creating the four-degree-of-freedom system was to verify the matrix operations and the FORTRAN program. The weighting matrices that were tried and accepted in methods 1 and 2 of this study apply to trusses in general, and they may or may not be applicable to other structural systems in addition to the four-degree-of-freedom system.

**TABLE II**  
**TEST DAMPING MATRIX OF THE FOUR-DEGREE-OF-FREEDOM SYSTEM**

1-MODE CORRELATION								
Real				Imaginary				
Column 1	Column 2	Column 3	Column 4	Column 1	Column 2	Column 3	Column 4	
Row 1	2.00E+00	-1.00E+00	-6.25E-04	-1.09E-03	-6.44E-08	-1.29E-07	2.16E-07	5.83E-07
Row 2	-1.00E+00	2.00E+00	-1.00E+00	-3.71E-03	-1.29E-07	-1.34E-07	1.25E-06	2.88E-06
Row 3	-6.25E-04	-1.00E+00	2.00E+00	-1.01E+00	2.16E-07	1.25E-06	4.58E-06	9.19E-06
Row 4	-1.09E-03	-3.71E-03	-1.01E+00	9.89E-01	5.83E-07	2.88E-06	9.19E-06	1.81E-05

2-MODE CORRELATION								
Real				Imaginary				
Column 1	Column 2	Column 3	Column 4	Column 1	Column 2	Column 3	Column 4	
Row 1	1.99E+00	-1.02E+00	-7.03E-03	1.77E-02	1.73E-05	2.51E-05	-1.46E-05	-1.12E-05
Row 2	-1.02E+00	1.95E+00	-1.02E+00	4.74E-02	2.52E-05	9.23E-06	-5.94E-05	2.66E-05
Row 3	-7.03E-03	-1.02E+00	1.99E+00	-9.88E-01	-1.38E-05	-5.75E-05	-4.04E-05	6.61E-05
Row 4	1.77E-02	4.74E-02	-9.88E-01	9.42E-01	-9.19E-06	3.22E-05	6.99E-05	-7.25E-05

**TABLE III**

**EIGENVALUES OF FOUR-DEGREE-OF-FREEDOM SYSTEM  
1-MODE CORRELATION  
METHOD 1**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	1.22E+01
2	XXX	XXX
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.08E-02	1.22E+01
2	XXX	XXX
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	-1.22E+01
2	XXX	XXX
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE IV**

**EIGENVELOCITIES OF FOUR-DEGREE-OF-FREEDOM SYSTEM  
1-MODE CORRELATION  
METHOD 1**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	-3.75E-03	1.95E+00	-4.16E-03	1.95E+00	-3.99E-03	-1.95E+00
2	1	-7.94E-03	4.73E+00	-8.83E-03	4.73E+00	-8.41E-03	-4.73E+00
3	1	-9.17E-03	9.30E+00	-1.02E-02	9.30E+00	-9.36E-03	-9.30E+00
4	1	-9.75E-03	1.22E+01	-1.08E-02	1.22E+01	-9.75E-03	-1.22E+01

**TABLE V**

**EIGENVALUES OF FOUR-DEGREE-OF-FREEDOM SYSTEM  
2-MODE CORRELATION  
METHOD 1**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	1.22E+01
2	-7.08E-02	-3.28E+01
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.08E-02	1.22E+01
2	-7.86E-02	-3.28E+01
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	1.22E+01
2	-7.08E-02	-3.28E+01
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE VI**

**EIGENVELOCITIES OF FOUR-DEGREE-OF-FREEDOM SYSTEM  
2-MODE CORRELATION  
METHOD 1**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	-3.74E-03	1.94E+00	-4.16E-03	1.94E+00	-3.78E-03	1.95E+00
1	2	-2.14E-03	-6.08E+00	-2.38E-03	-6.08E+00	-2.14E-03	-6.08E+00
2	1	-7.94E-03	4.71E+00	-8.82E-03	4.71E+00	-7.98E-03	4.73E+00
2	2	9.22E-03	-1.19E+01	1.02E-02	-1.19E+01	9.22E-03	-1.19E+01
3	1	-9.19E-03	9.26E+00	-1.02E-02	9.26E+00	-9.03E-03	9.30E+00
3	2	2.42E-02	-5.68E+00	2.68E-02	-5.68E+00	2.42E-02	-5.68E+00
4	1	-9.78E-03	1.22E+01	-1.09E-02	1.22E+01	-9.75E-03	1.22E+01
4	2	-3.68E-03	7.89E+00	-4.09E-03	7.89E+00	-3.68E-03	7.89E+00



**TABLE VII**

**EIGENVALUES OF FOUR DEGREE-OF-FREEDOM SYSTEM  
3-MODE CORRELATION  
METHOD 1**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	1.22E+01
2	-7.08E-02	-3.28E+01
3	-7.08E-02	3.28E+01
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.08E-02	1.22E+01
2	-7.86E-02	-3.28E+01
3	-7.86E-02	3.28E+01
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-9.75E-03	1.22E+01
2	2.41E+00	-3.27E+01
3	2.36E+00	3.27E+01
4	XXX	XXX
5	XXX	XXX

**TABLE VIII**

**EIGENVELOCITIES OF FOUR DEGREE OF FREEDOM SYSTEM  
3-MODE CORRELATION  
METHOD 1**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	-3.74E-03	1.93E+00	-4.16E-03	1.93E+00	-3.74E-03	1.93E+00
1	2	-2.09E-03	-6.06E+00	-2.32E-03	-6.06E+00	5.08E-01	-8.69E-04
1	3	1.36E-02	-6.06E+00	1.51E-02	-6.06E+00	5.11E-01	3.40E-04
2	1	-7.95E-03	4.69E+00	-8.83E-03	4.69E+00	-7.95E-03	4.69E+00
2	2	9.28E-03	-1.19E+01	1.03E-02	-1.19E+01	1.00E+00	0.00E+00
2	3	1.33E-02	-1.19E+01	1.47E-02	-1.19E+01	1.00E+00	0.00E+00
3	1	-9.23E-03	9.23E+00	-1.03E-02	9.23E+00	-9.23E-03	9.23E+00
3	2	2.41E-02	-5.66E+00	2.68E-02	-5.66E+00	4.84E-01	3.36E-03
3	3	-1.34E-02	-5.66E+00	-1.49E-02	-5.66E+00	4.68E-01	5.82E-05
4	1	-9.84E-03	1.21E+01	-1.09E-02	1.21E+01	-9.84E-03	1.21E+01
4	2	-3.73E-03	7.86E+00	-4.14E-03	7.86E+00	-6.66E-01	-7.49E-04
4	3	-1.12E-02	7.86E+00	-1.24E-02	7.86E+00	-1.12E-02	7.86E+00

## 4.6 12-BAR Truss

The finite element model of the 12-BAR truss is shown in figures 3 and 4. The MSC/NASTRAN finite element program was used to model the 12-BAR truss. As can be observed in figure 3, the 12-BAR plane truss consists of three bays. Each bay of the truss consists of two longerons, one batten, and one diagonal. The numbers in figure 3 represent the finite element node numbers. Nodes 1, 5, 9, 10, 11, 12, 13, 14, 15, and 16 have been grounded. The remaining nodes can translate in two orthogonal directions in the plane of the figure. We can see in figure 4 that there are 12 ROD elements, which have been designated by the abbreviation, RD. ROD elements in MSC/NASTRAN are general purpose elements with tension and compression capability. Therefore, the stiffness matrix of a ROD element is the same as the stiffness matrix of a simple spring. The mass of a ROD element is divided by two and the masses are placed on the two respective nodes surrounding the ROD element. Also, in figure 4 the viscous dampers (VISC) have been designated by the abbreviation, VS. The purpose of these elements is to resist the dynamic motion of the connecting joint in each translational direction. Thus, they represent the viscous damping in the joints of the truss. The VISC elements in the MSC/NASTRAN finite element program are general purpose elements and they represent the general viscous dampers. In other words, the force produced by an element is equal to the velocity of the connecting degree-of-freedom times a damping coefficient. Since the lateral vibration motion of the 12-BAR truss is dominant, only one viscous element has been attached to every joint of the truss, except to the joints at the free end of the truss, where two viscous dampers per joint have been attached. There are two viscous dampers per joint at the free end of the truss to resist the dynamic motion in two directions (lateral and axial) simultaneously, if a small coupling between the axial vibration modes and the lateral vibration modes exists.

The ROD elements are made of steel. ROD elements of the analytical 12-BAR truss finite element model have the following properties:

$$\text{Modulus of elasticity} = E = 30.0E6 \frac{\text{lb}}{\text{in}^2}$$

$$\text{Poisson's ratio} = \nu = .33$$

$$\text{Outer diameter of truss members} = O.D. = 1.0 \text{ in}$$

$$\text{Thickness of truss members} = t = 0.075 \text{ in}$$

$$\text{Mass density of truss members} = \rho = 0.0283 \frac{\text{lb} - \text{sec}^2}{\text{in}}$$

The VISC elements of the analytical 12-BAR truss finite element model have the following property:

$$\text{Damping coefficient} = C = 1.0 \frac{\text{lb} - \text{sec}}{\text{in}}$$

It should be noted that the selected value of the damping coefficient is quite large, and based on this selection, the real and imaginary parts of the eigenvalues and eigenvectors have the same order of magnitude. This selection determines whether the methodology and the FORTRAN program in this study can support and handle structures with large damping.

In an effort to visualize the vibration motion of the analytical finite element model of the 12-BAR truss, the complex eigenvalue–eigenvector extraction of MSC/NASTRAN was activated, and the modal deformation plots have been included here. The modal deformation plots for the first four modes of the 12-BAR truss when it contains the analytical spatial properties are found in figures 5 through 12. Figures 5 through 8 present the real portion of the modal deformations, and figures 9 through 12 represent the imaginary portion of the modal deformations. In order to better visualize and understand the motion of the 12-BAR truss, the modal deformations have been superimposed on the undeformed shape of the truss. We can see in figures 5 through 8 that the modal deformation plots represent the system level modes, where the entire structure is in motion. It should be noted that the kinetic energy values associated with modes 2 through 4 are close, since the shape of their deformation and their respective normalized eigenvectors are close.

The following steps were taken to construct a test model:

- 1) The coefficients of the VISC elements 17, 18, 19, and 20 were varied. When varying the damping coefficients of these elements, 50 percent of the elements of the damping matrix in the test model have different values than those in the analytical model.
- 2) The modulus of elasticity of the diagonal elements, namely ROD elements 7, 9, and 11 were varied. When varying the modulus of elasticity, the stiffness of the ROD elements would vary proportionally. In general, when varying the stiffness values of diagonal elements of a truss the bending stiffness is greatly influenced since a large portion of the bending stiffness is due to the action (tension and compression) in the diagonal elements.

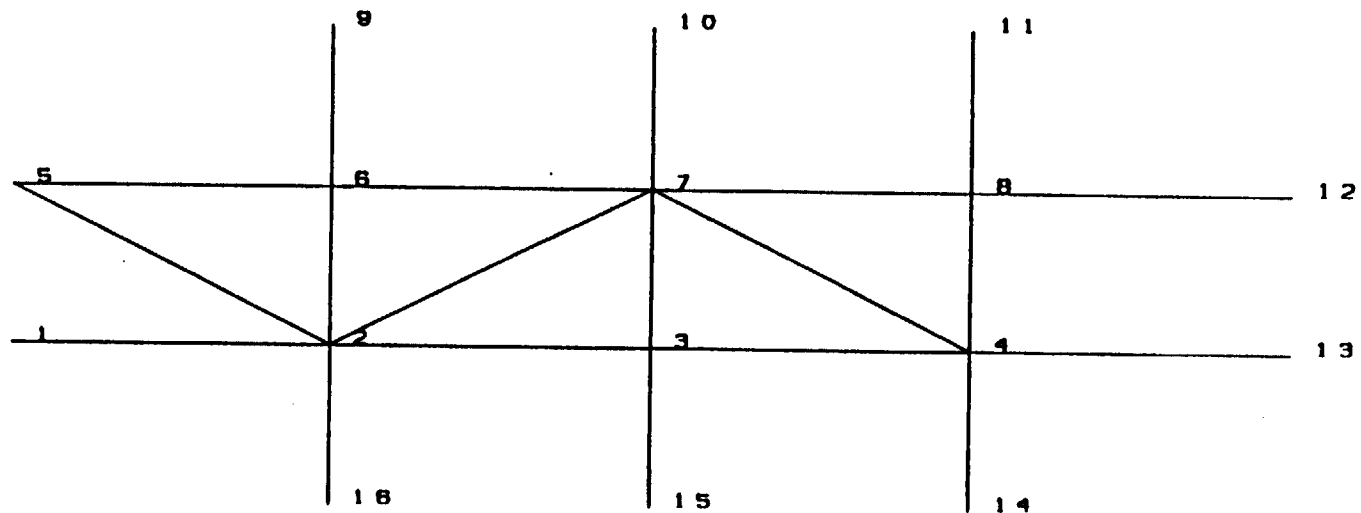
Variance of the properties of the diagonal ROD elements was not done randomly. Because of the connectivity of the diagonals to the joints, and the fact that they connect to the joint at an angle, the effective stiffness of the joint combined with that of the diagonal may be different from the effective stiffness of the joint combined with that of the longerons or of the battens. Selection of the VISC elements was done

randomly by the author who assumed that the manufacturing process for the joints might have been different enough to create different damping coefficients for the truss joints. However, in a practical structural system, the difference between the analytical spatial properties and the test spatial properties cannot be as severe as 10 percent of 50 percent of the joints. The 5 percent or 10 percent reduction of the analytical spatial properties was used here only to examine the robustness of the methodology of this study.

In an effort to examine the accuracy and robustness of the methodology of the system identification and the FORTRAN program used for this study, the values, as described above, were varied twice. The first variation of the values was a 5 percent reduction of the analytical model's properties. All the simulation computer runs for this 5 percent reduction were called 95 models. The second variation of the values was a 10 percent reduction of the analytical model's properties. All of the simulation computer runs for this 10 percent reduction were called 90 models.

In an effort to visualize the vibration motion of the test finite element model of the 12-BAR truss, the complex eigenvalue-eigenvector extraction of MSC/NASTRAN was again activated. The modal deformation plots have been included here for the modal deformation of the first four modes of the 12-BAR truss, 95 model, in figures 13 through 20. The modal deformation plots of the 90 model have not been included because the deformation shapes are the same as the ones of the 95 model. When comparing the modal deformation plots of the 90 model with those of the analytical model, the modal deformation of the first, second and fourth eigenvalues have the same shape. The modal deformation plots of the third eigenvalue appear to be different. However, when the third modal deformation shapes of the two finite element models are examined visually, we can see that the modal deformations are the same. They are, in fact, mirror images of each other, and the only difference is the way they have been plotted by the MSC/NASTRAN. As can be observed from the modal deformation plots in figures 13 through 20, the modes are system level modes, where the entire structure is in motion.

After the simulation computer runs were completed, the test modal data were saved on tape for the system identification process. Then the system identification was begun by exercising method 1 and method 2 presented in chapter III. The results of two-mode to five-mode system identification for the 95 model have been presented and discussed. However, the results of only the two-mode and five-mode system identifications for the 90 model have been presented here due to the similarity in results of the two models. The system identification for the two-mode computer runs use modes 1 and 2. The system identification for the five-mode computer runs use modes 1, 3, 5, 7, and 9. The motivation for using a wide variety of modes in identifying the 95 and 90 models was to examine the degradation in the quality of the test eigenvalues and eigenvectors, if any.



**Figure 3: Analysis Finite Element Model Of 12-BAR Truss, Node Numbers**

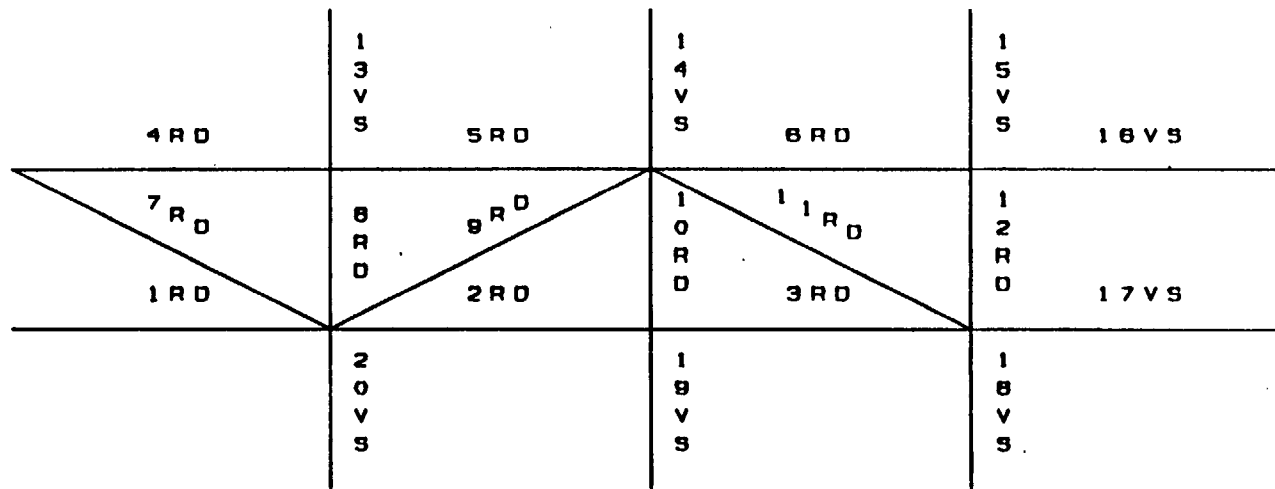
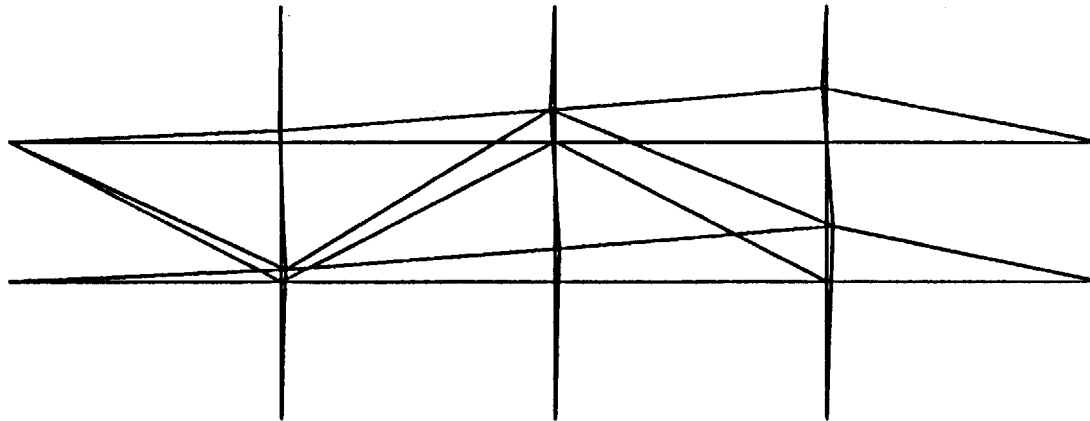
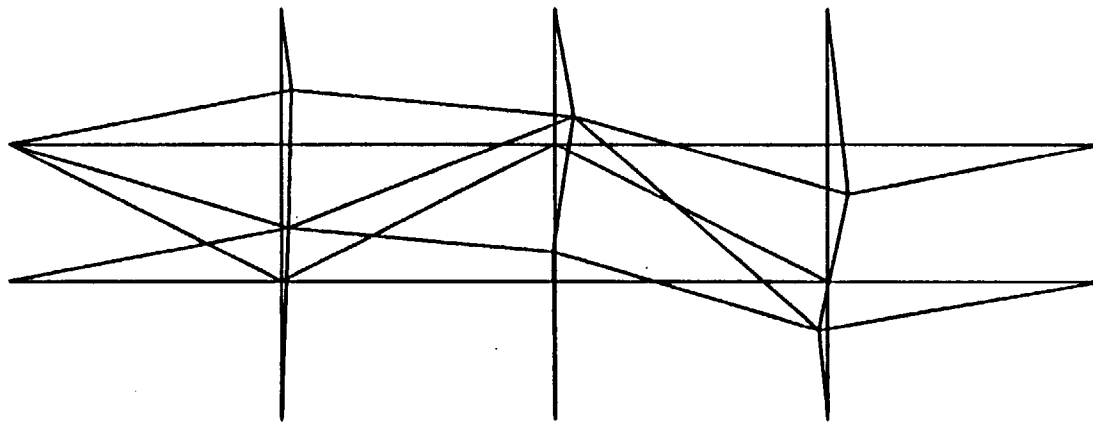


Figure 4: Analysis Finite Element Model Of 12-BAR Truss, Element Numbers

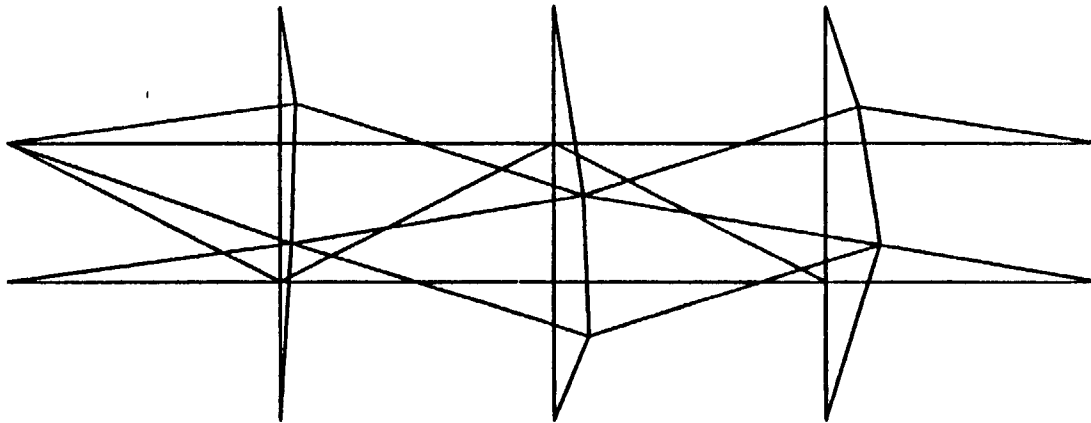


**Figure 5: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 1, Frequency=162.5463 Hz, Phase=0**

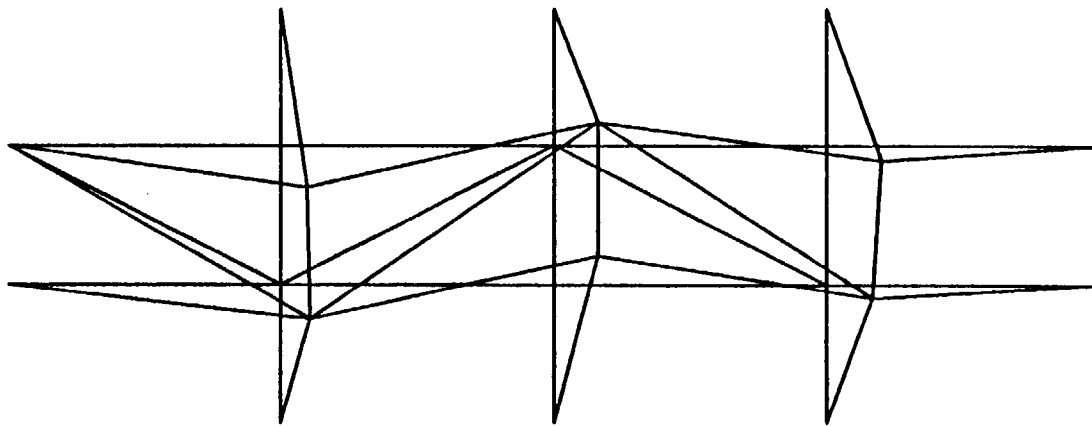




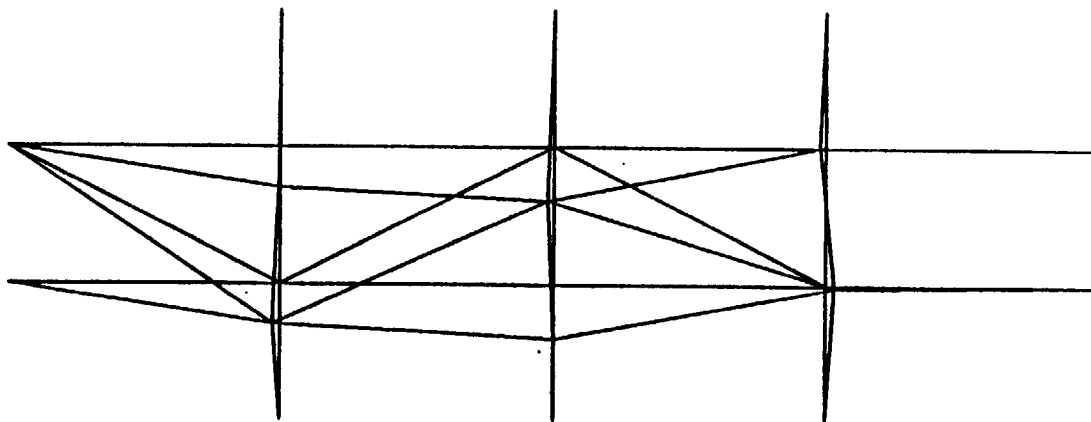
**Figure 6: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 2, Frequency=609.2841 Hz, Phase=0**



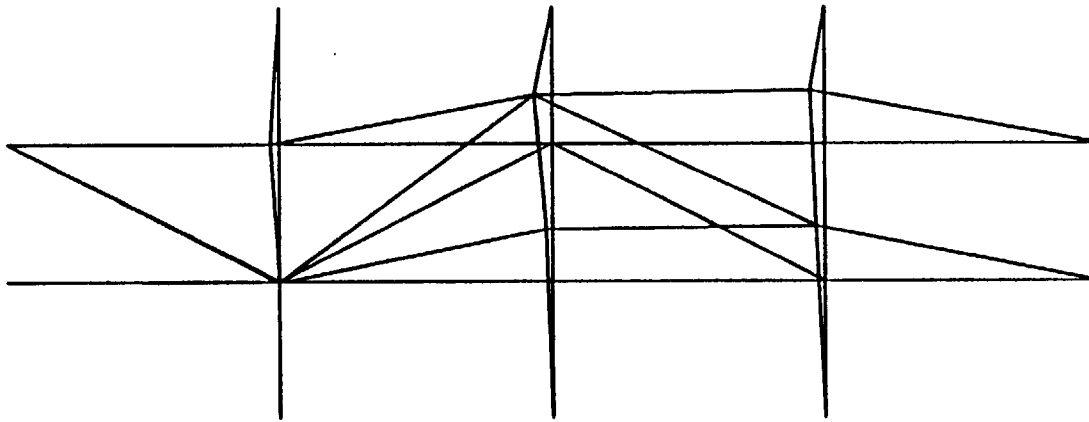
**Figure 7: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 3, Frequency=945.7958 Hz, Phase=0**



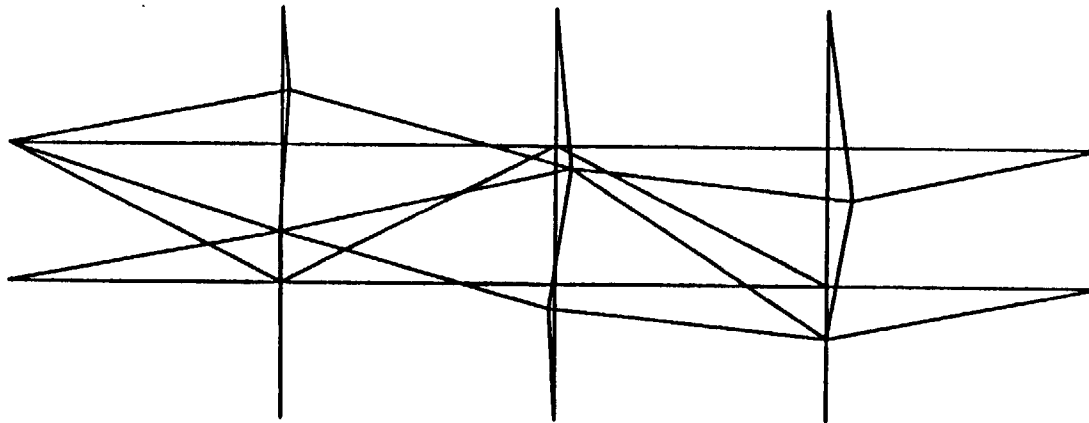
**Figure 8: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 4, Frequency=1596.589 Hz, Phase=0**



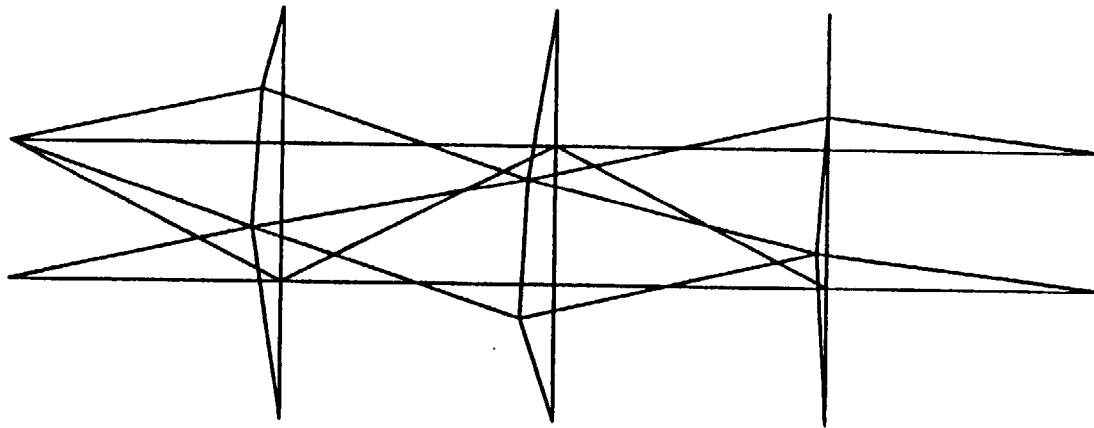
**Figure 9: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 1, Frequency=162.5463 Hz, Phase=-90**



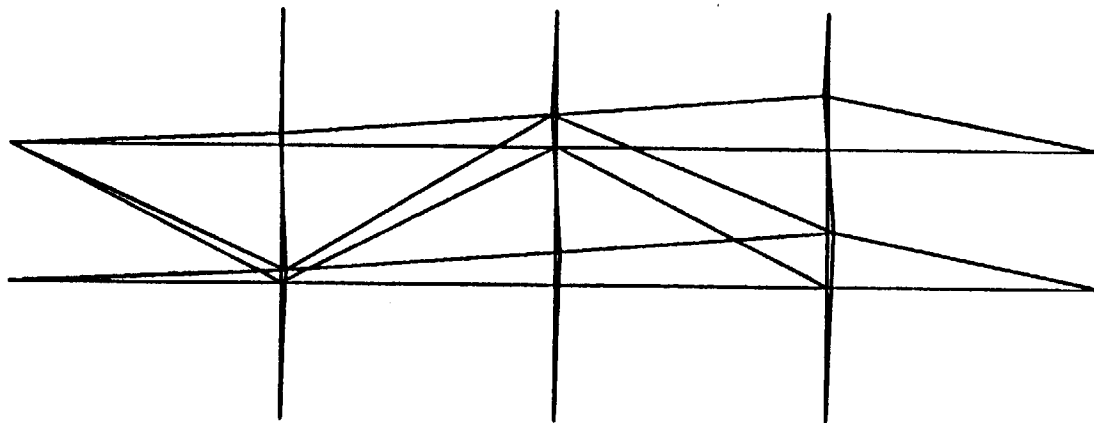
**Figure 10: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 2, Frequency=609.2841 Hz, Phase=-90**



**Figure 11: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 3, Frequency=945.7958 Hz, Phase=-90**

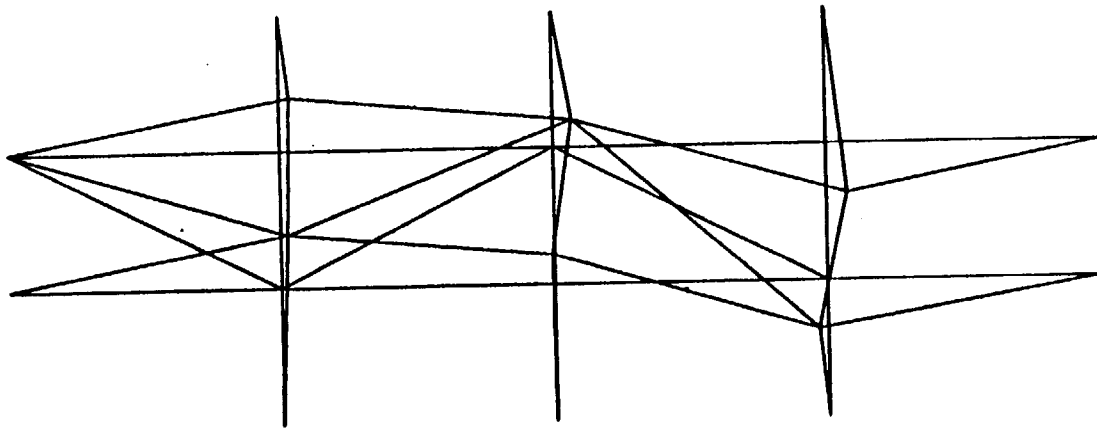


**Figure 12: Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss,  
Mode 4, Frequency=1596.589 Hz, Phase=-90**

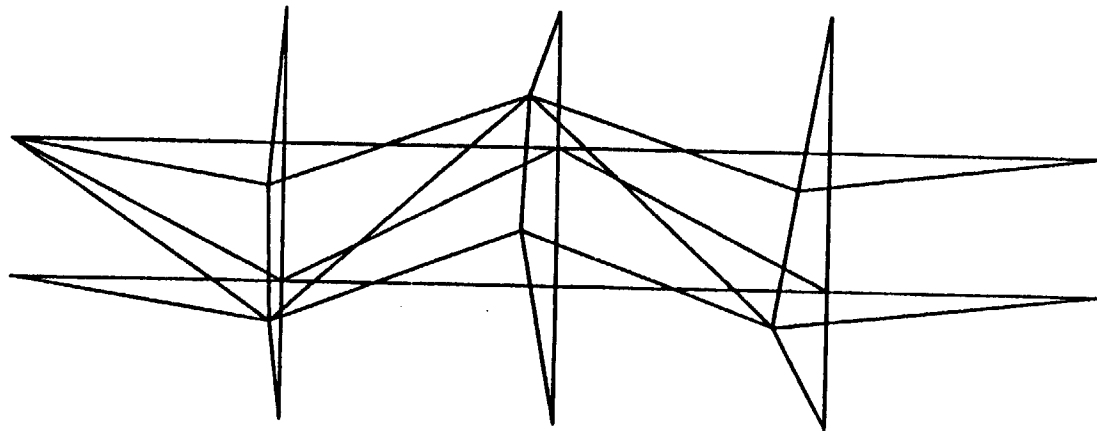


**Figure 13: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 1, Frequency=162.6529 Hz, Phase=0**

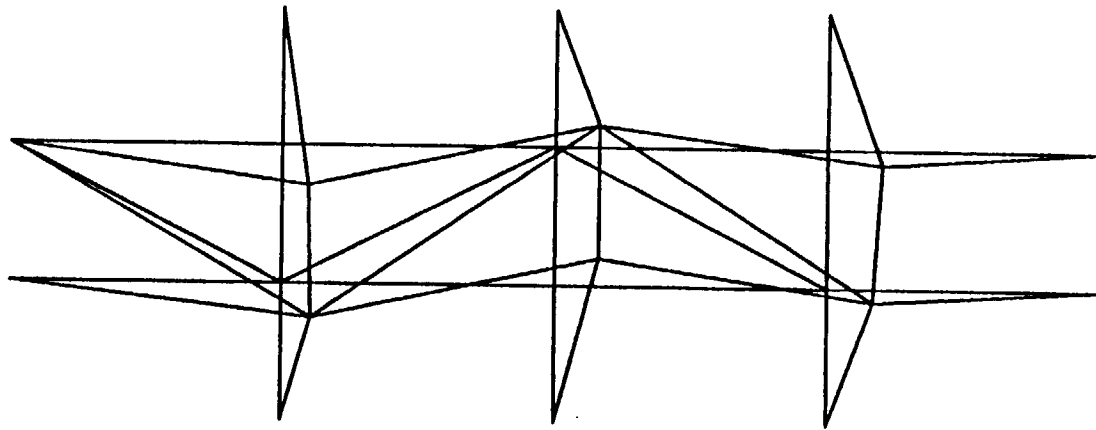




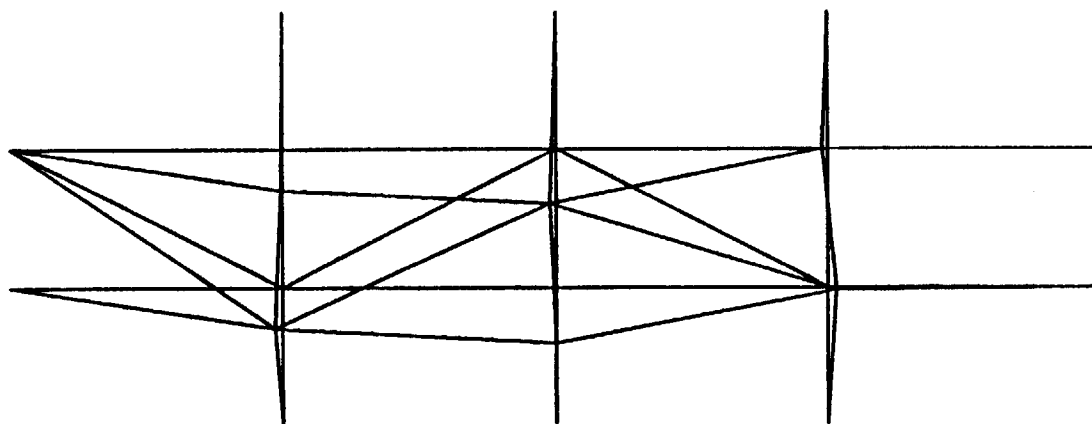
**Figure 14: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 2, Frequency=604.2818 Hz, Phase=0**



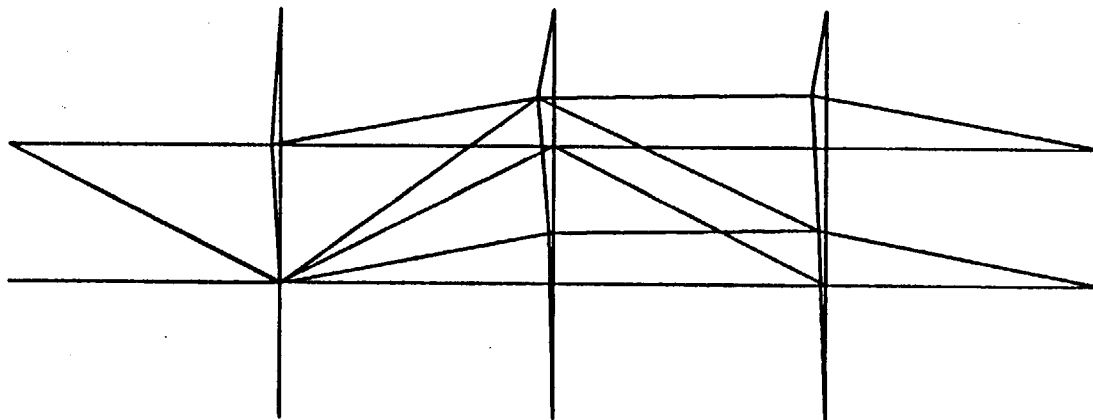
**Figure 15: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 3, Frequency=940.3127 Hz, Phase=0**



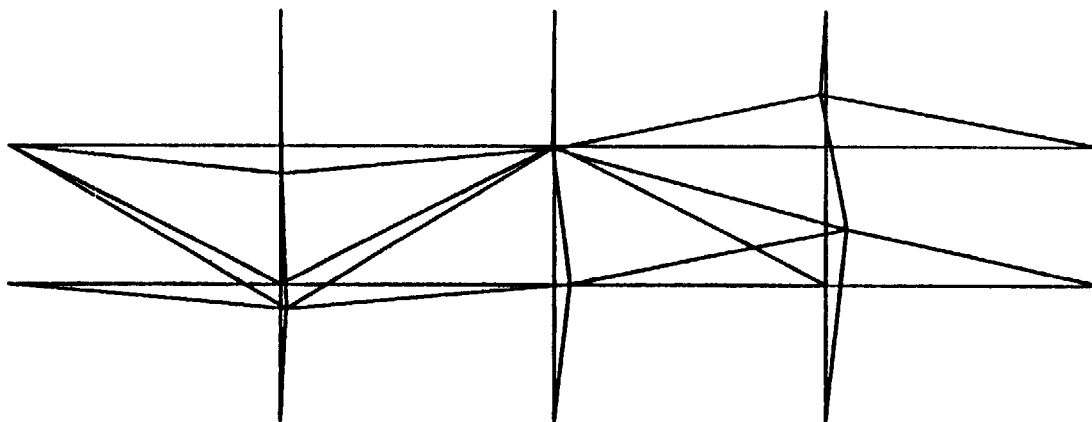
**Figure 16: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 4, Frequency=1581.193 Hz, Phase=0**



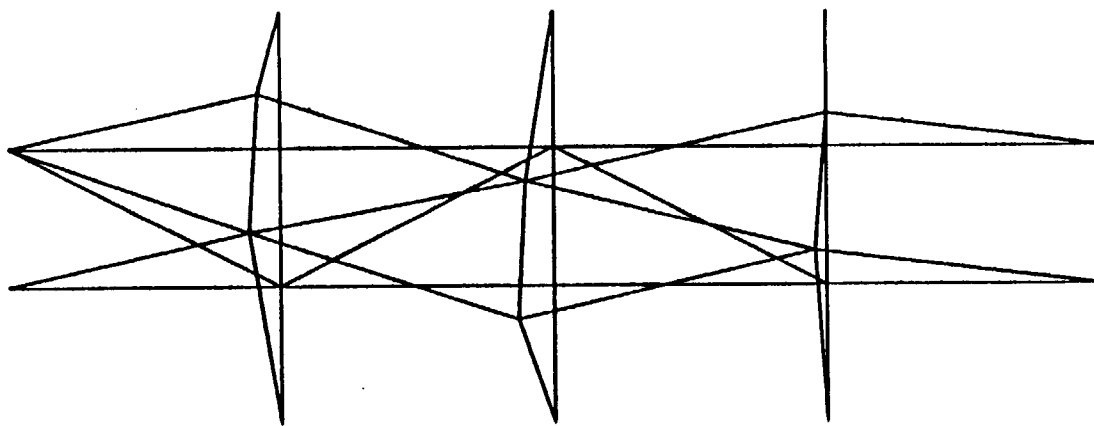
**Figure 17: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 1, Frequency=162.6529 Hz, Phase=-90**



**Figure 18: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 2, Frequency=604.2818 Hz, Phase=-90**



**Figure 19: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 3, Frequency=940.3127 Hz, Phase=-90**



**Figure 20: Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss,  
95 Model  
Mode 4, Frequency=1581.193 Hz, Phase=-90**

#### 4.6.1 Results for 12-BAR Truss, 95 Model, Method 1

**1) Method 1, Two-Mode System Identification:** The results of this computer run are documented in figures 21 through 25 and in tables IX and X. We can see in table IX that the improvements of the first and second eigenvalues are nearly 100 percent. Also, the eigenvelocities in table X show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first and the second modes. The values of the orthogonality condition matrix, equation (3.11), are plotted in figures 21 and 22. The values in figure 21 are based on the analytical eigensystem vectors and the analytical spatial property matrices. The values in figure 22 are based on the test eigensystem vectors and the analytical spatial property matrices. The off-diagonal terms in figures 21 and 22 are nearly zero, since method 1 only improves upon the damping matrix. The values of the orthogonality condition matrix (equation (3.10)), are plotted in figures 23 through 25. The values in figure 23 are based on the test modal properties and the analytical spatial property matrices. The values in figure 24 are based on the test modal properties and the test spatial property matrices. Figure 25 values are based on the correlated modal properties and the test spatial property matrices. As can be observed in figures 23 through 25, the off-diagonal terms are not quite zero. Non-zero off-diagonal terms were expected in figure 23 because, theoretically speaking, the test modal properties belong to a structural system that is different from the analytical finite element model. Although it is hard to visualize, the off-diagonal terms have improved in figure 25 and they have gotten closer to zero. In addition to the improvement in the values of the eigenvalues and eigenvectors, one of the measures used to determine the success of the system identification process is to examine the orthogonality condition before and after the system identification. If the off-diagonal terms of the orthogonality condition have approached zero, the system identification is successful. In fact, examining the orthogonality matrices in large structures is the only measure of checking the success of the system identification process for the eigenvelocities, since the large number of DOF's of the system makes it impractical to check the eigenvelocities. When the eigenvelocities in table X are visually examined the values of the orthogonality condition matrices in figures 24 and 25 are approximately the same, since the correlated eigenvelocities have approached the test eigenvelocities. Overall, the results of the two-mode system identification appear quite satisfactory.



**TABLE IX****EIGENVALUES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 1, 95 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-461.705	1021.985
2	-377.578	-3796.814
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

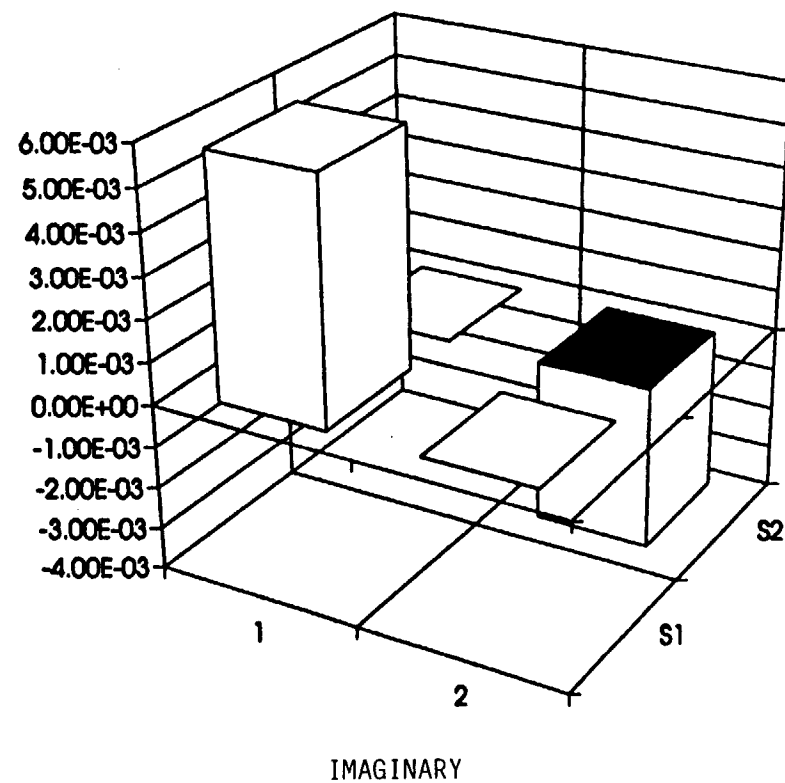
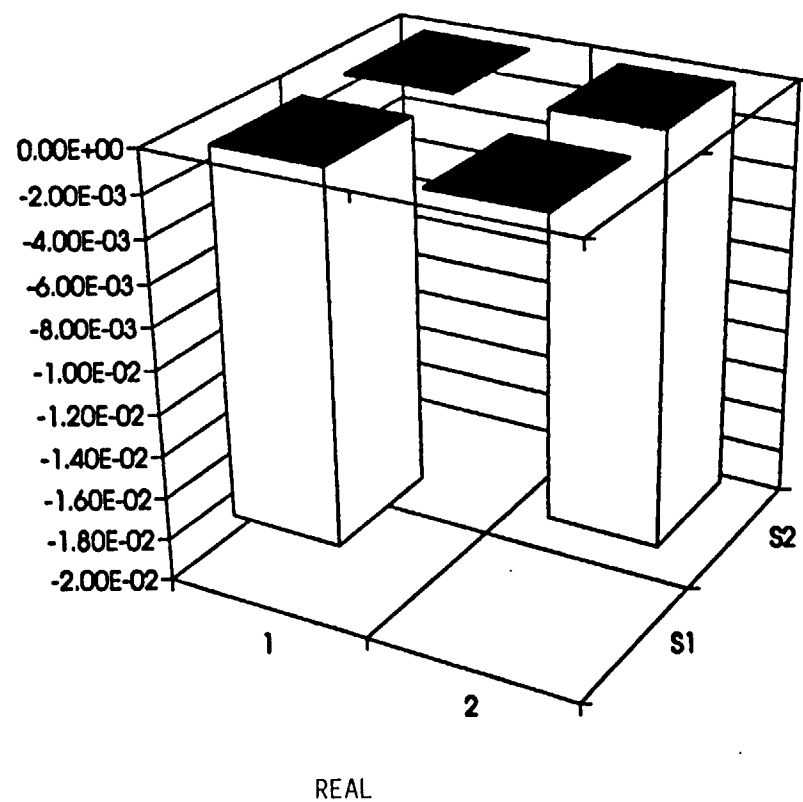
ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-473.899	1021.307
2	-386.132	-3828.244
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-461.7066	1021.891
2	-377.562	-3796.294
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

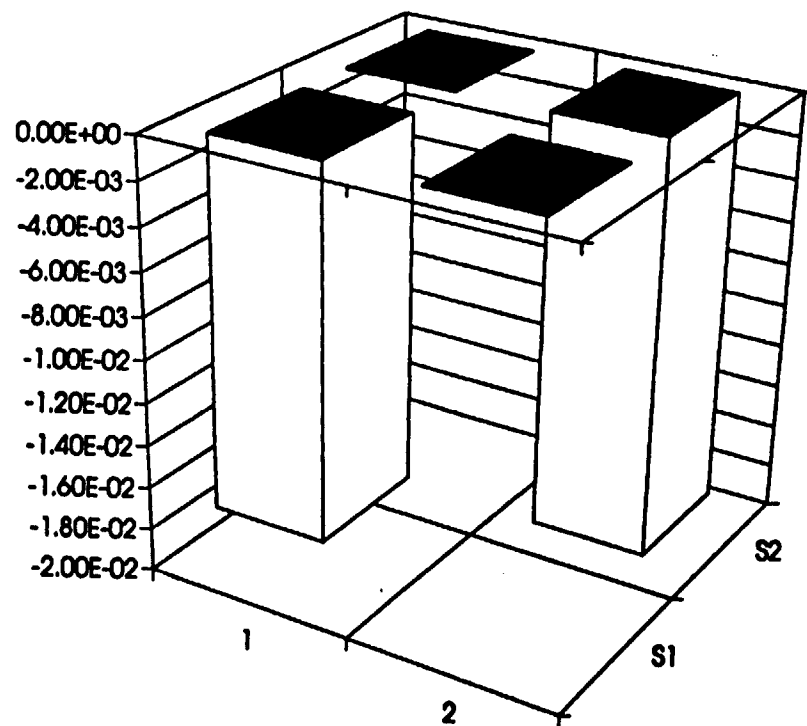
**TABLE X**

**EIGENVELOCITIES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD1, 95 MODEL**

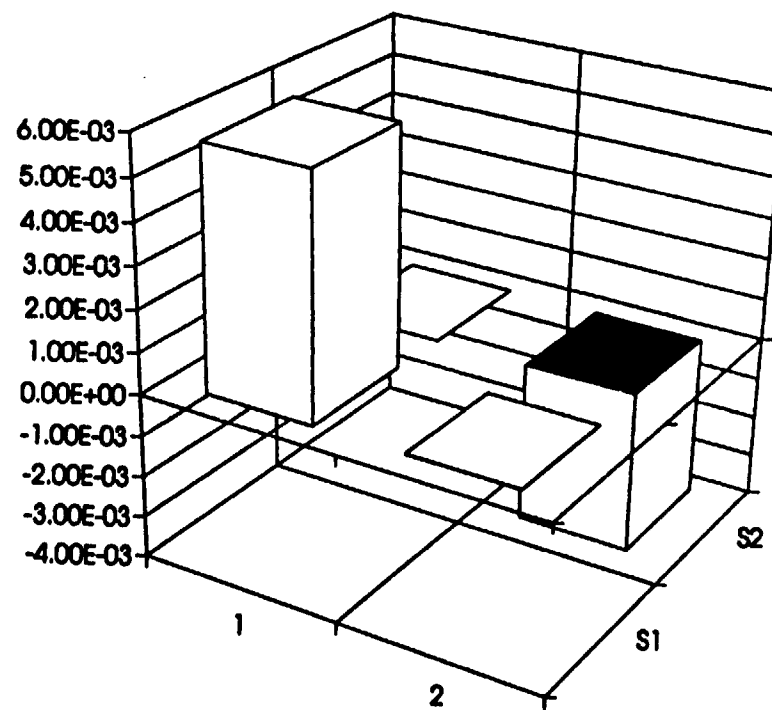
D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	7.03E-02	1.91E-03	7.07E-02	1.95E-03	7.10E-02	1.95E-03
1	2	-1.35E-01	-9.26E-03	-1.41E-01	-9.50E-03	-1.40E-01	-9.27E-03
2	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
2	2	-1.09E+00	-7.51E-02	-1.09E+00	-7.58E-02	-1.08E+00	-7.42E-02
3	1	8.46E-02	-5.33E-05	8.51E-02	-6.00E-05	8.52E-02	-6.52E-05
3	2	2.26E-02	7.22E-03	1.96E-02	7.77E-03	1.98E-02	7.73E-03
4	1	6.03E-01	1.47E-02	6.02E-01	1.47E-02	6.03E-01	1.47E-02
4	2	-5.86E-01	-1.06E-01	-5.91E-01	-1.08E-01	-5.87E-01	-1.07E-01
5	1	9.86E-02	-2.26E-03	9.93E-02	-2.32E-03	9.92E-02	-2.33E-03
5	2	1.80E-01	2.36E-02	1.79E-01	2.49E-02	1.79E-01	2.46E-02
6	1	9.99E-01	6.37E-04	9.99E-01	6.14E-04	9.99E-01	6.15E-04
6	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
7	1	-4.00E-02	7.12E-04	-4.03E-02	7.13E-04	-4.01E-02	7.10E-04
7	2	-2.00E-01	-2.32E-03	-2.04E-01	-2.13E-03	-2.03E-01	-2.00E-03
8	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
8	2	-1.11E+00	-7.81E-02	-1.11E+00	-7.89E-02	-1.11E+00	-7.72E-02
9	1	-7.99E-02	1.54E-03	-8.05E-02	1.54E-03	-8.00E-02	1.54E-03
9	2	-3.92E-01	-6.29E-03	-3.98E-01	-6.00E-03	-3.97E-01	-5.72E-03
10	1	6.02E-01	1.46E-02	6.00E-01	1.47E-02	6.02E-01	1.47E-02
10	2	-5.73E-01	-1.03E-01	-5.78E-01	-1.05E-01	-5.74E-01	-1.03E-01
11	1	-8.01E-02	1.65E-03	-8.07E-02	1.65E-03	-8.03E-02	1.64E-03
11	2	-4.03E-01	-8.92E-03	-4.09E-01	-8.63E-03	-4.08E-01	-8.32E-03
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.01E+00	3.04E-03	1.01E+00	3.00E-03	1.01E+00	2.96E-03



**Figure 21: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.11),  
Analysis Model  
2 Mode System Identification**

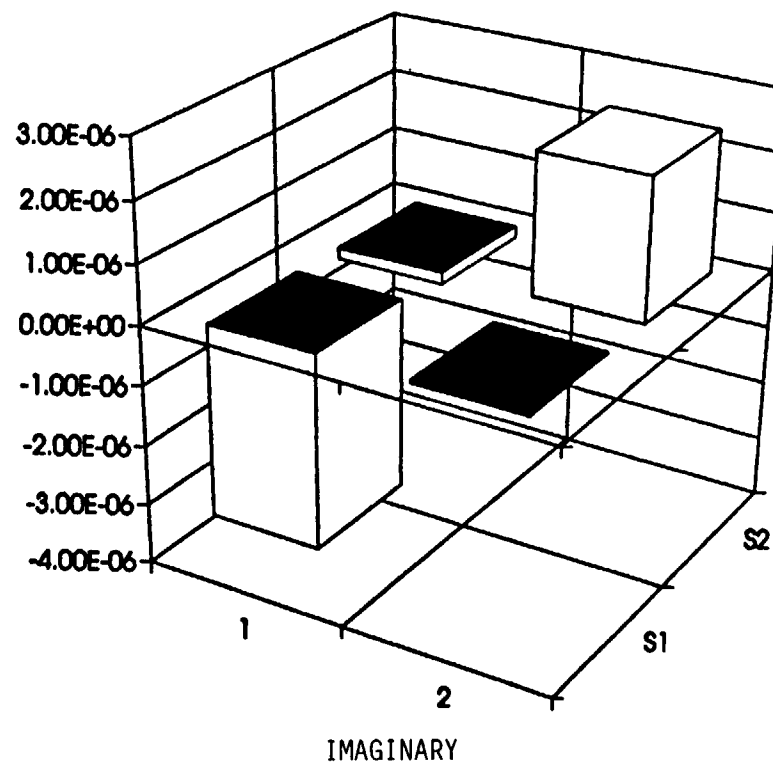
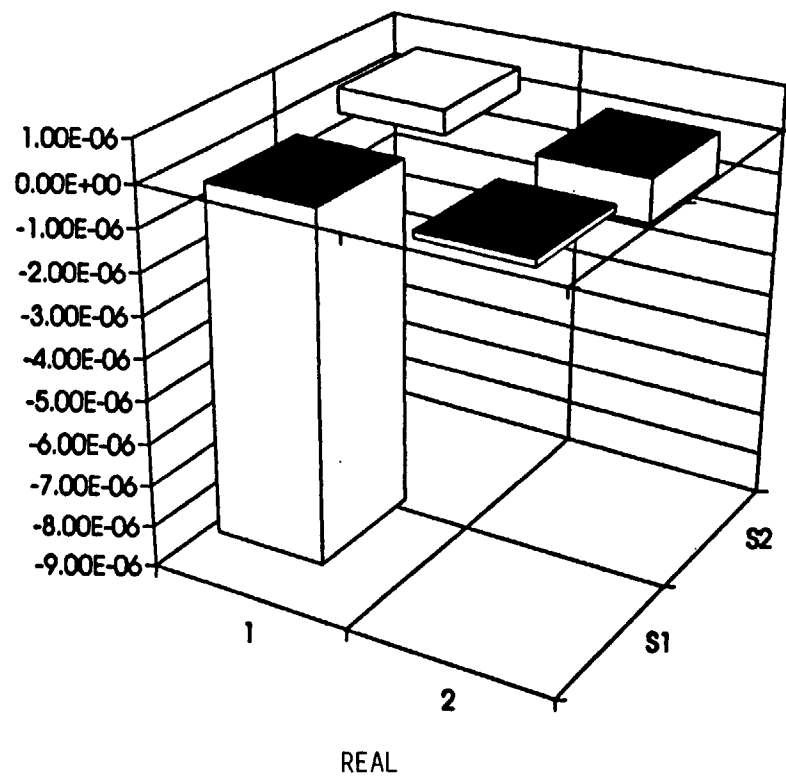


REAL

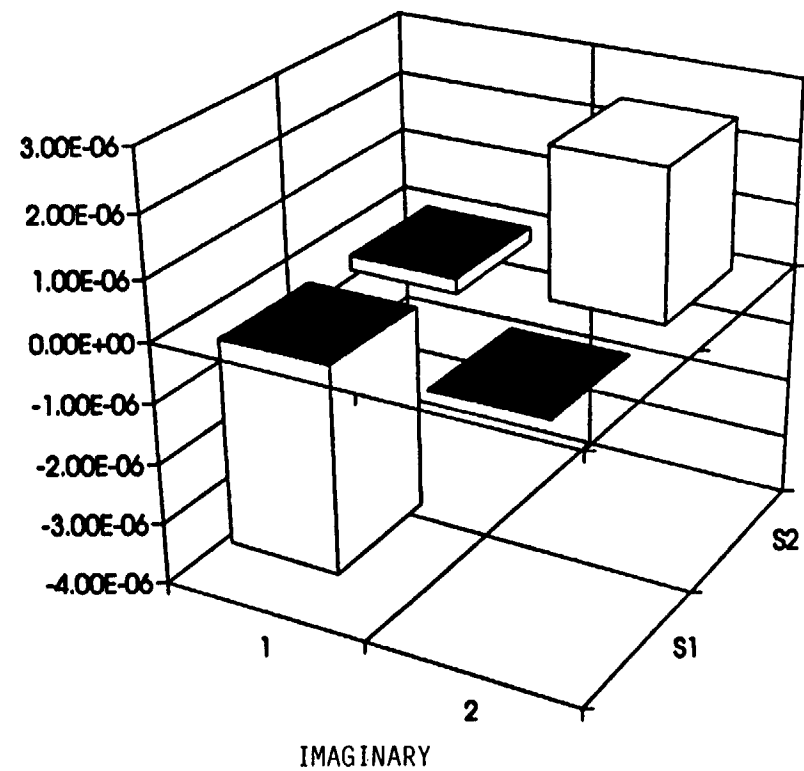
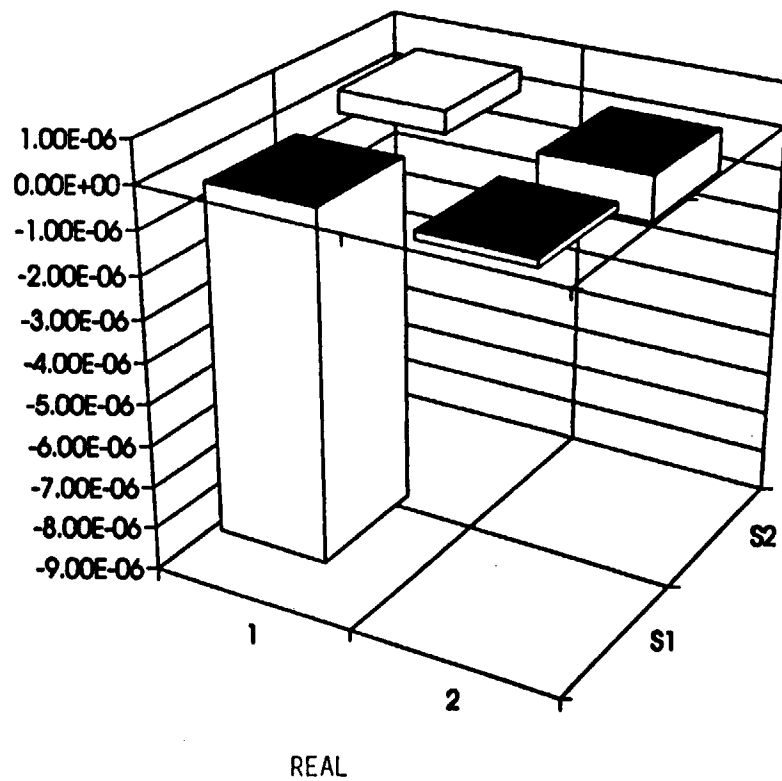


IMAGINARY

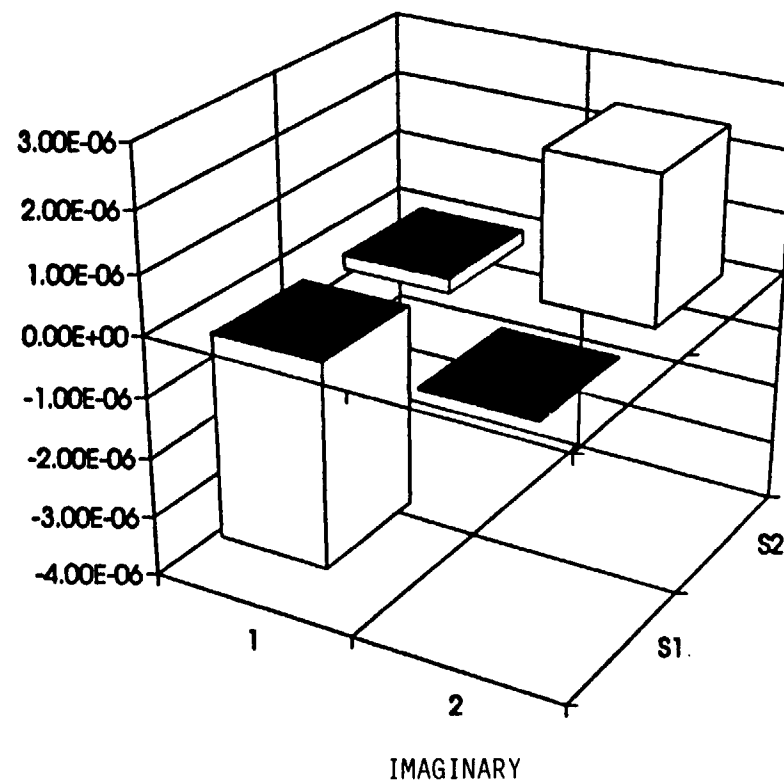
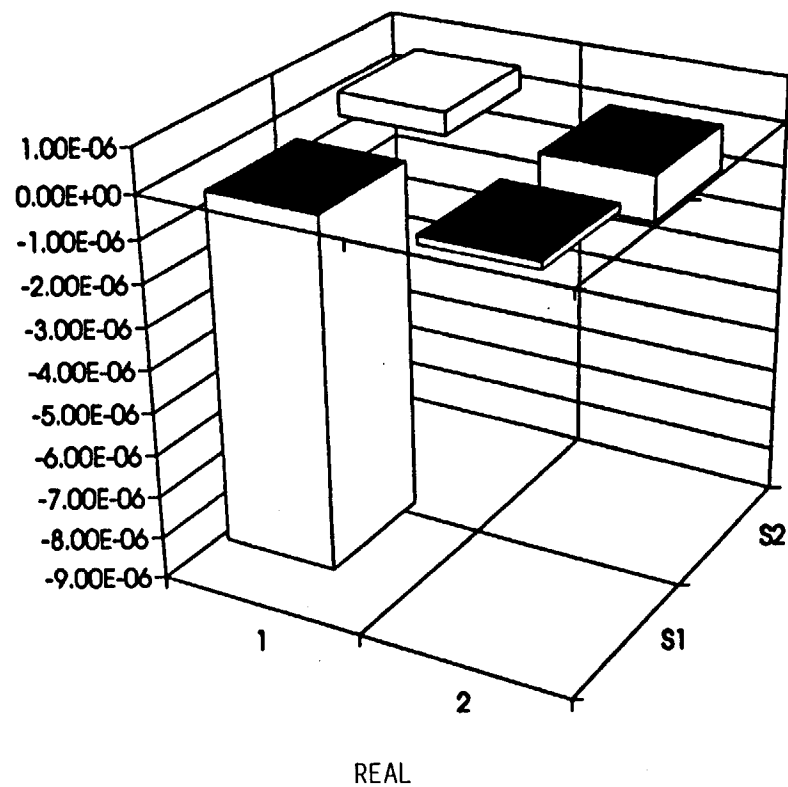
**Figure 22: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.11),  
Test Model, 95 Model  
2 Mode System Identification**



**Figure 23: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10),  
Test Modal Properties, and Analytical Spatial Properties  
2 Mode System Identification Using Method 1**



**Figure 24: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10),  
Test Modal Properties, and Test Spatial Properties  
2 Mode System Identification Using Method 1**



**Figure 25: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10),  
Correlated Modal Properties, and Test Spatial Properties  
2 Mode System Identification Using Method 1**

**2) Method 1, Three-Mode System Identification:** The results of this computer run are documented in tables XI and XII. Table XI indicates that the improvements of the first, second, and third eigenvalues are 100 percent. Also, the eigenvelocities in table XII show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first, second, and third modes. Overall, the results of the three-mode system identification appear quite satisfactory.

**3) Method 1, Four-Mode System Identification:** The results of this computer run are documented in tables XIII and XIV. Table XIII indicates that the improvements of the first, second, third, and fourth eigenvalues are 100 percent. Also, the eigenvelocities in table XIV show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first, second, third, and fourth modes. Overall, the results of the four-mode system identification appear quite satisfactory.

**4) Method 1, Five-Mode System Identification:** The results of this computer run are documented in figure 26, and in tables XV and XVI. Table XV indicates that the real portions of the first, second, and third have improved somewhat, but not nearly as much as with the two-mode to four-mode system identification. The real portions of the fourth and fifth modes did not improve, and in fact they became worse than the analytical eigenvalues. The imaginary portions of all eigenvalues did not improve at all. Also, the eigenvelocities in table XVI show the same divergence, that is, the magnitudes of nearly all eigenvelocities either remained the same or worsened. The values of the orthogonality condition matrix (equation (3.10)) have been plotted in figure 26 with the values based upon the correlated modal properties and the test spatial property matrices. We can see in figure 26 that the off-diagonal terms are not near zero, and in fact some of their values are very close to the diagonal terms. Therefore, the method 1 system identification using five modes failed.



**TABLE XI**

**EIGENVALUES OF 12-BAR TRUSS  
3-MODE CORRELATION  
METHOD 1, 95 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	-5.94E+03
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	XXX	XXX
5	XXX	XXX

**TABLE XII**

**EIGENVELOCITIES OF 12-BAR TRUSS  
3-MODE CORRELATION  
METHOD 1, 95 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	7.03E-02	1.91E-03	7.07E-02	1.95E-03	7.10E-02	1.96E-03
1	2	-1.35E-01	-9.26E-03	-1.41E-01	-9.50E-03	-1.40E-01	-9.52E-03
1	3	-2.14E-01	7.74E-03	-2.19E-01	7.51E-03	-2.19E-01	7.72E-03
2	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
2	2	-1.09E+00	-7.51E-02	-1.09E+00	-7.58E-02	-1.08E+00	-7.53E-02
2	3	-7.17E-01	-3.10E-02	-7.11E-01	-3.32E-02	-7.13E-01	-3.13E-02
3	1	8.46E-02	-5.33E-05	8.51E-02	-6.00E-05	8.51E-02	-4.15E-05
3	2	2.26E-02	7.22E-03	1.96E-02	7.77E-03	2.03E-02	7.22E-03
3	3	-6.38E-01	2.21E-02	-6.52E-01	2.22E-02	-6.52E-01	2.23E-02
4	1	6.03E-01	1.47E-02	6.02E-01	1.47E-02	6.04E-01	1.46E-02
4	2	-5.86E-01	-1.06E-01	-5.91E-01	-1.08E-01	-5.88E-01	-1.06E-01
4	3	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
5	1	9.86E-02	-2.26E-03	9.93E-02	-2.32E-03	9.90E-02	-2.29E-03
5	2	1.80E-01	2.36E-02	1.79E-01	2.49E-02	1.80E-01	2.39E-02
5	3	-9.94E-01	2.64E-02	-1.01E+00	2.64E-02	-1.02E+00	2.65E-02
6	1	9.99E-01	6.37E-04	9.99E-01	6.14E-04	9.99E-01	6.14E-04
6	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
6	3	-6.77E-01	7.04E-02	-6.77E-01	7.26E-02	-6.78E-01	7.07E-02
7	1	-4.00E-02	7.12E-04	-4.03E-02	7.13E-04	-4.01E-02	7.21E-04
7	2	-2.00E-01	-2.32E-03	-2.04E-01	-2.13E-03	-2.03E-01	-2.34E-03
7	3	-2.83E-01	5.86E-04	-2.90E-01	2.69E-04	-2.91E-01	6.90E-04
8	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
8	2	-1.11E+00	-7.81E-02	-1.11E+00	-7.89E-02	-1.11E+00	-7.84E-02
8	3	-7.58E-01	-3.49E-02	-7.52E-01	-3.71E-02	-7.54E-01	-3.52E-02

# CONTINUATION OF TABLE XII

## EIGENVELOCITIES OF 12-BAR TRUSS 3-MODE CORRELATION METHOD 1, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
9	1	-7.99E-02	1.54E-03	-8.05E-02	1.54E-03	-8.02E-02	1.56E-03
9	2	-3.92E-01	-6.29E-03	-3.98E-01	-6.00E-03	-3.97E-01	-6.38E-03
9	3	-5.36E-01	-2.30E-03	-5.49E-01	-3.09E-03	-5.50E-01	-2.26E-03
10	1	6.02E-01	1.46E-02	6.00E-01	1.47E-02	6.02E-01	1.46E-02
10	2	-5.73E-01	-1.03E-01	-5.78E-01	-1.05E-01	-5.74E-01	-1.03E-01
10	3	9.46E-01	-2.23E-03	9.46E-01	-2.53E-03	9.46E-01	-2.50E-03
11	1	-8.01E-02	1.65E-03	-8.07E-02	1.65E-03	-8.04E-02	1.66E-03
11	2	-4.03E-01	-8.92E-03	-4.09E-01	-8.63E-03	-4.08E-01	-9.01E-03
11	3	-5.73E-01	-8.67E-03	-5.88E-01	-9.61E-03	-5.88E-01	-8.71E-03
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.01E+00	3.04E-03	1.01E+00	3.00E-03	1.01E+00	2.95E-03
12	3	-7.00E-01	6.91E-02	-7.01E-01	7.14E-02	-7.01E-01	6.95E-02

**TABLE XIII**

**EIGENVALUES OF 12-BAR TRUSS  
4-MODE CORRELATION  
METHOD 1, 95 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	-2.48E+02	-9.93E+03
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	-5.94E+03
4	-2.55E+02	-1.00E+04
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	-2.48E+02	-9.93E+03
5	XXX	XXX

**TABLE XIV**

**EIGENVELOCITIES OF 12-BAR TRUSS  
4-MODE CORRELATION  
METHOD 1, 95 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	7.03E-02	1.91E-03	7.07E-02	1.95E-03	7.09E-02	1.95E-03
1	2	-1.35E-01	-9.26E-03	-1.41E-01	-9.50E-03	-1.38E-01	-9.55E-03
1	3	-2.14E-01	7.74E-03	-2.19E-01	7.51E-03	-2.11E-01	7.67E-03
1	4	5.42E-01	-2.27E-02	5.43E-01	-2.29E-02	5.42E-01	-2.25E-02
2	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
2	2	-1.09E+00	-7.51E-02	-1.09E+00	-7.58E-02	-1.09E+00	-7.50E-02
2	3	-7.17E-01	-3.10E-02	-7.11E-01	-3.32E-02	-7.17E-01	-3.09E-02
2	4	-6.12E-01	3.92E-02	-6.26E-01	4.03E-02	-6.10E-01	3.88E-02
3	1	8.46E-02	-5.33E-05	8.51E-02	-6.00E-05	8.49E-02	-4.59E-05
3	2	2.26E-02	7.22E-03	1.96E-02	7.77E-03	2.38E-02	7.18E-03
3	3	-6.38E-01	2.21E-02	-6.52E-01	2.22E-02	-6.37E-01	2.22E-02
3	4	8.11E-01	-2.59E-02	8.10E-01	-2.56E-02	8.11E-01	-2.54E-02
4	1	6.03E-01	1.47E-02	6.02E-01	1.47E-02	6.03E-01	1.46E-02
4	2	-5.86E-01	-1.06E-01	-5.91E-01	-1.08E-01	-5.86E-01	-1.06E-01
4	3	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
4	4	5.01E-01	-2.52E-02	5.13E-01	-2.55E-02	5.01E-01	-2.47E-02
5	1	9.86E-02	-2.26E-03	9.93E-02	-2.32E-03	9.88E-02	-2.30E-03
5	2	1.80E-01	2.36E-02	1.79E-01	2.49E-02	1.84E-01	2.39E-02
5	3	-9.94E-01	2.64E-02	-1.01E+00	2.64E-02	-9.98E-01	2.65E-02
5	4	8.37E-01	-9.05E-03	8.28E-01	-7.89E-03	8.32E-01	-7.93E-03
6	1	9.99E-01	6.37E-04	9.99E-01	6.14E-04	9.99E-01	6.14E-04
6	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
6	3	-6.77E-01	7.04E-02	-6.77E-01	7.26E-02	-6.76E-01	7.05E-02
6	4	-2.30E-01	2.56E-02	-2.35E-01	2.57E-02	-2.29E-01	2.53E-02

# CONTINUATION OF TABLE XIV

## EIGENVELOCITIES OF 12-BAR TRUSS 2-MODE CORRELATION METHOD 1, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
7	1	-4.00E-02	7.12E-04	-4.03E-02	7.13E-04	-4.02E-02	7.18E-04
7	2	-2.00E-01	-2.32E-03	-2.04E-01	-2.13E-03	-2.01E-01	-2.34E-03
7	3	-2.83E-01	5.86E-04	-2.90E-01	2.69E-04	-2.82E-01	7.20E-04
7	4	4.82E-01	-1.61E-02	4.81E-01	-1.63E-02	4.81E-01	-1.62E-02
8	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.28E-01	1.08E-02
8	2	-1.11E+00	-7.81E-02	-1.11E+00	-7.89E-02	-1.11E+00	-7.81E-02
8	3	-7.58E-01	-3.49E-02	-7.52E-01	-3.71E-02	-7.59E-01	-3.47E-02
8	4	-7.22E-01	4.01E-02	-7.40E-01	4.15E-02	-7.23E-01	4.00E-02
9	1	-7.99E-02	1.54E-03	-8.05E-02	1.54E-03	-8.03E-02	1.55E-03
9	2	-3.92E-01	-6.29E-03	-3.98E-01	-6.00E-03	-3.93E-01	-6.36E-03
9	3	-5.36E-01	-2.30E-03	-5.49E-01	-3.09E-03	-5.36E-01	-2.12E-03
9	4	8.19E-01	-2.02E-02	8.15E-01	-2.01E-02	8.16E-01	-2.00E-02
10	1	6.02E-01	1.46E-02	6.00E-01	1.47E-02	6.02E-01	1.46E-02
10	2	-5.73E-01	-1.03E-01	-5.78E-01	-1.05E-01	-5.73E-01	-1.03E-01
10	3	9.46E-01	-2.23E-03	9.46E-01	-2.53E-03	9.45E-01	-2.50E-03
10	4	4.25E-01	-2.46E-02	4.34E-01	-2.51E-02	4.23E-01	-2.43E-02
11	1	-8.01E-02	1.65E-03	-8.07E-02	1.65E-03	-8.06E-02	1.66E-03
11	2	-4.03E-01	-8.92E-03	-4.09E-01	-8.63E-03	-4.04E-01	-8.95E-03
11	3	-5.73E-01	-8.67E-03	-5.88E-01	-9.61E-03	-5.72E-01	-8.31E-03
11	4	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.01E+00	3.04E-03	1.01E+00	3.00E-03	1.01E+00	2.96E-03
12	3	-7.00E-01	6.91E-02	-7.01E-01	7.14E-02	-7.00E-01	6.93E-02
12	4	-2.53E-01	2.53E-02	-2.59E-01	2.55E-02	-2.53E-01	2.50E-02

**TABLE XV**

**EIGENVALUES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD1, 95 MODEL**

<b>TEST EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	-2.48E+02	-9.93E+03
5	-2.90E+02	1.74E+04

<b>ANALYSIS EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	-5.94E+03
4	-2.55E+02	-1.00E+04
5	-2.96E+02	1.76E+04

<b>CORRELATED EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.65E+02	-1.03E+03
2	-3.84E+02	3.86E+03
3	-3.36E+02	5.98E+03
4	-2.57E+02	1.01E+04
5	-2.96E+02	-1.78E+04

**TABLE XVI**

**EIGENVELOCITIES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 1, 95 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	7.03E-02	1.91E-03	7.07E-02	1.95E-03	7.08E-02	-2.04E-03
1	2	-1.35E-01	-9.26E-03	-1.41E-01	-9.50E-03	1.29E-01	1.15E-03
1	3	-2.14E-01	7.74E-03	-2.19E-01	7.51E-03	2.20E-01	2.12E-03
1	4	5.42E-01	-2.27E-02	5.43E-01	-2.29E-02	5.44E-01	2.43E-02
1	5	-3.06E-01	-1.68E-02	-3.01E-01	-1.74E-02	-3.01E-01	1.86E-02
2	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.25E-01	-9.73E-03
2	2	-1.09E+00	-7.51E-02	-1.09E+00	-7.58E-02	9.78E-01	1.22E-03
2	3	-7.17E-01	-3.10E-02	-7.11E-01	-3.32E-02	6.83E-01	-5.10E-02
2	4	-6.12E-01	3.92E-02	-6.26E-01	4.03E-02	-6.42E-01	-4.22E-02
2	5	-4.70E-02	-4.60E-03	-4.60E-02	-4.59E-03	-4.62E-02	4.44E-03
3	1	8.46E-02	-5.33E-05	8.51E-02	-6.00E-05	8.56E-02	-4.99E-04
3	2	2.26E-02	7.22E-03	1.96E-02	7.77E-03	-1.51E-02	6.75E-03
3	3	-6.38E-01	2.21E-02	-6.52E-01	2.22E-02	6.46E-01	5.43E-03
3	4	8.11E-01	-2.59E-02	8.10E-01	-2.56E-02	8.09E-01	2.79E-02
3	5	-7.94E-01	-3.26E-02	-7.92E-01	-3.42E-02	-7.92E-01	3.61E-02
4	1	6.03E-01	1.47E-02	6.02E-01	1.47E-02	6.01E-01	-1.37E-02
4	2	-5.86E-01	-1.06E-01	-5.91E-01	-1.08E-01	5.38E-01	-5.72E-02
4	3	1.00E+00	0.00E+00	1.00E+00	0.00E+00	-9.69E-01	2.55E-02
4	4	5.01E-01	-2.52E-02	5.13E-01	-2.55E-02	5.25E-01	2.74E-02
4	5	-6.83E-02	3.39E-03	-6.82E-02	3.64E-03	-6.84E-02	-3.55E-03
5	1	9.86E-02	-2.26E-03	9.93E-02	-2.32E-03	9.99E-02	1.71E-03
5	2	1.80E-01	2.36E-02	1.79E-01	2.49E-02	-1.58E-01	1.19E-02
5	3	-9.94E-01	2.64E-02	-1.01E+00	2.64E-02	1.00E+00	0.00E+00
5	4	8.37E-01	-9.05E-03	8.28E-01	-7.89E-03	8.24E-01	9.33E-03



# CONTINUATION OF TABLE XVI

## EIGENVELOCITIES OF 12-BAR TRUSS

### 5-MODE CORRELATION

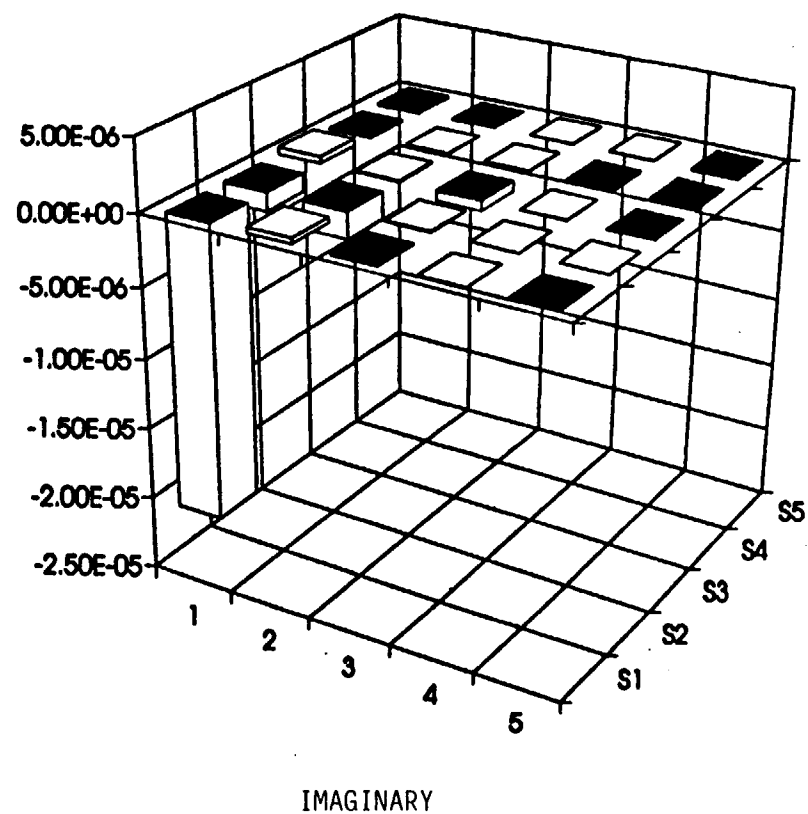
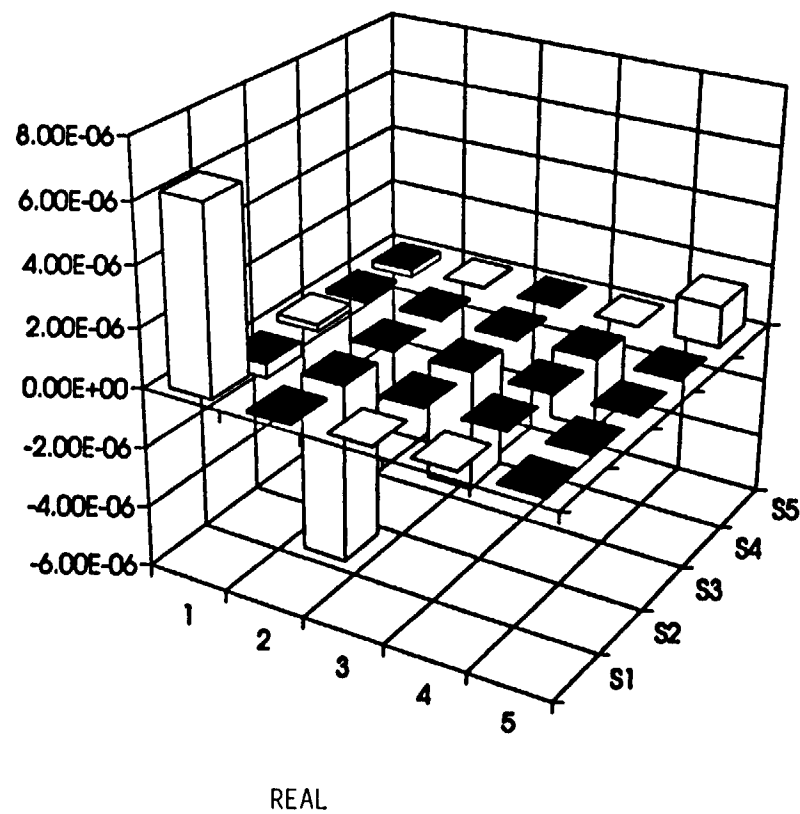
### METHOD 1, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	-5.50E-01	6.21E-03	-5.37E-01	6.36E-03	-5.37E-01	-6.42E-03
6	1	9.99E-01	6.37E-04	9.99E-01	6.14E-04	9.99E-01	-6.52E-04
6	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	-8.94E-01	-6.03E-02
6	3	-6.77E-01	7.04E-02	-6.77E-01	7.26E-02	6.60E-01	5.12E-02
6	4	-2.30E-01	2.56E-02	-2.35E-01	2.57E-02	-2.40E-01	-2.72E-02
6	5	2.23E-01	1.70E-02	2.20E-01	1.70E-02	2.21E-01	-1.73E-02
7	1	-4.00E-02	7.12E-04	-4.03E-02	7.13E-04	-4.05E-02	-4.70E-04
7	2	-2.00E-01	-2.32E-03	-2.04E-01	-2.13E-03	1.85E-01	1.12E-02
7	3	-2.83E-01	5.86E-04	-2.90E-01	2.69E-04	2.88E-01	-6.68E-03
7	4	4.82E-01	-1.61E-02	4.81E-01	-1.63E-02	4.81E-01	1.67E-02
7	5	4.12E-01	4.22E-02	4.09E-01	4.32E-02	4.08E-01	-4.55E-02
8	1	2.28E-01	1.08E-02	2.26E-01	1.08E-02	2.25E-01	-9.74E-03
8	2	-1.11E+00	-7.81E-02	-1.11E+00	-7.89E-02	1.00E+00	0.00E+00
8	3	-7.58E-01	-3.49E-02	-7.52E-01	-3.71E-02	7.21E-01	-5.59E-02
8	4	-7.22E-01	4.01E-02	-7.40E-01	4.15E-02	-7.58E-01	-4.36E-02
8	5	-8.76E-02	-6.89E-03	-8.74E-02	-7.06E-03	-8.75E-02	7.01E-03
9	1	-7.99E-02	1.54E-03	-8.05E-02	1.54E-03	-8.07E-02	-1.23E-03
9	2	-3.92E-01	-6.29E-03	-3.98E-01	-6.00E-03	3.61E-01	2.02E-02
9	3	-5.36E-01	-2.30E-03	-5.49E-01	-3.09E-03	5.44E-01	-1.62E-02
9	4	8.19E-01	-2.02E-02	8.15E-01	-2.01E-02	8.14E-01	2.08E-02
9	5	4.44E-01	3.27E-02	4.33E-01	3.26E-02	4.33E-01	-3.40E-02
10	1	6.02E-01	1.46E-02	6.00E-01	1.47E-02	6.00E-01	-1.37E-02
10	2	-5.73E-01	-1.03E-01	-5.78E-01	-1.05E-01	5.25E-01	-5.53E-02
10	3	9.46E-01	-2.23E-03	-2.59E-01	2.55E-02	-9.17E-01	2.16E-02

# CONTINUATION OF TABLE XVI

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 1, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	4.25E-01	-2.46E-02	4.34E-01	-2.51E-02	4.45E-01	2.69E-02
10	5	-3.67E-02	1.22E-03	-3.60E-02	1.24E-03	-3.62E-02	-1.01E-03
11	1	-8.01E-02	1.65E-03	-8.07E-02	1.65E-03	-8.12E-02	-1.06E-03
11	2	-4.03E-01	-8.92E-03	-4.09E-01	-8.63E-03	3.73E-01	1.85E-02
11	3	-5.73E-01	-8.67E-03	-5.88E-01	-9.61E-03	5.84E-01	-2.36E-02
11	4	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
11	5	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.01E+00	3.04E-03	1.01E+00	3.00E-03	-9.06E-01	-5.86E-02
12	3	-7.00E-01	6.91E-02	-7.01E-01	7.14E-02	6.82E-01	4.94E-02
12	4	-2.53E-01	2.53E-02	-2.59E-01	2.55E-02	-2.64E-01	-2.71E-02
12	5	3.09E-01	1.65E-02	3.08E-01	1.67E-02	3.09E-01	-1.71E-02



**Figure 26: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10),  
Correlated Modal Properties, and Test Spatial Properties  
5 Mode System Identification Of 95 Model Using Method 1**

#### 4.6.2 Results for 12-BAR Truss, 95 Model, Method 2

Although the system identification for the 90 model has been performed with the entire range of available modes (two to five modes), only the lower and upper bonds of the these computer runs will be presented here.

**1) Method 2, Two-Mode System Identification:** The results of this computer run are documented in tables XVII and XVIII. Table XVII indicates that the improvements of the first and second eigenvalues are 100 percent. Also, the eigenvelocities in table XVIII show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first and second modes. Overall, the results of the two-mode system identification appear quite satisfactory.

**2) Method 2, Five-Mode System Identification:** The results of this computer run are documented in figures 27 and 28, and in tables XIX and XX. Table XIX indicates that the improvements of the first, second, third, fourth, and fifth eigenvalues are 100 percent. Also, the eigenvelocities in table XX show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first, second, third, fourth, and fifth modes. Because both the stiffness and damping matrices are correlated in method 2, the orthogonality conditions based on equations (3.10) and (3.11) are presented pictorially. Figure 27 shows the orthogonality condition matrix based on equation (3.10) and figure 28 shows the orthogonality condition matrix based on equation (3.11). According to these figures all the off-diagonal terms are near zero, except for one term, 1-S2. If smaller off-diagonal terms in the orthogonality condition matrices are desired, then either the tolerances for the convergence criteria in method 2 should be lessened, or a convergence criterion based on off-diagonal terms should be added. However, the down side of lessening the tolerances for the convergence criteria, or adding a convergence criterion for the off-diagonal terms of the orthogonality condition matrices, requires more iterations and longer computer usage time. Overall, the results of the five-mode system identification appear quite satisfactory.

**TABLE XVII**

**EIGENVALUES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 2, 95 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

TABLE XVIII

**EIGENVELOCITIES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 2, 95 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	1.28E-01	8.47E-03	1.27E-01	8.25E-03	1.28E-01	8.47E-03
1	2	-1.69E-01	-8.56E-03	-1.62E-01	-8.42E-03	-1.68E-01	-9.02E-03
2	1	4.10E-01	3.53E-02	4.11E-01	3.51E-02	4.12E-01	3.54E-02
2	2	-1.30E+00	-6.91E-02	-1.30E+00	-6.85E-02	-1.30E+00	-6.98E-02
3	1	1.55E-01	5.82E-03	1.53E-01	5.67E-03	1.54E-01	5.89E-03
3	2	2.36E-02	8.91E-03	2.72E-02	8.21E-03	2.37E-02	7.59E-03
4	1	1.09E+00	6.86E-02	1.09E+00	6.77E-02	1.09E+00	6.81E-02
4	2	-7.09E-01	-1.18E-01	-7.05E-01	-1.16E-01	-7.06E-01	-1.15E-01
5	1	1.81E-01	2.69E-03	1.79E-01	2.63E-03	1.80E-01	2.86E-03
5	2	2.15E-01	2.62E-02	2.16E-01	2.47E-02	2.15E-01	2.40E-02
6	1	1.81E+00	7.06E-02	1.81E+00	6.92E-02	1.81E+00	7.02E-02
6	2	1.20E+00	-1.99E-02	1.20E+00	-1.97E-02	1.20E+00	-2.00E-02
7	1	-7.32E-02	-1.51E-03	-7.24E-02	-1.44E-03	-7.26E-02	-1.40E-03
7	2	-2.44E-01	1.50E-03	-2.40E-01	1.17E-03	-2.43E-01	6.66E-04
8	1	4.10E-01	3.54E-02	4.12E-01	3.51E-02	4.12E-01	3.55E-02
8	2	-1.33E+00	-7.23E-02	-1.33E+00	-7.17E-02	-1.33E+00	-7.32E-02
9	1	-1.46E-01	-2.80E-03	-1.45E-01	-2.66E-03	-1.45E-01	-2.60E-03
9	2	-4.76E-01	7.31E-04	-4.70E-01	1.95E-04	-4.76E-01	-7.69E-04
10	1	1.09E+00	6.85E-02	1.09E+00	6.76E-02	1.09E+00	6.79E-02
10	2	-6.93E-01	-1.15E-01	-6.89E-01	-1.12E-01	-6.90E-01	-1.12E-01
11	1	-1.47E-01	-2.63E-03	-1.45E-01	-2.48E-03	-1.46E-01	-2.40E-03
11	2	-4.90E-01	-2.19E-03	-4.83E-01	-2.73E-03	-4.90E-01	-3.84E-03
12	1	1.82E+00	6.96E-02	1.81E+00	6.82E-02	1.81E+00	6.91E-02
12	2	1.21E+00	-1.65E-02	1.21E+00	-1.64E-02	1.21E+00	-1.65E-02

**TABLE XIX**

**EIGENVALUES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 95 MODEL**

<b>TEST EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	-2.48E+02	-9.93E+03
5	-2.90E+02	1.74E+04

<b>ANALYSIS EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	-5.94E+03
4	-2.55E+02	-1.00E+04
5	-2.96E+02	1.76E+04

<b>CORRELATED EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.62E+02	1.02E+03
2	-3.78E+02	-3.80E+03
3	-3.33E+02	-5.91E+03
4	-2.48E+02	-9.93E+03
5	-2.90E+02	1.74E+04

**TABLE XX**

**EIGENVELOCITIES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 95 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	2.07E-01	5.64E-03	2.09E-01	5.82E-03	2.08E-01	5.71E-03
1	2	-2.65E-01	-3.74E-03	-2.76E-01	-3.45E-03	-2.70E-01	-4.05E-03
1	3	-3.96E-01	6.87E-03	-4.03E-01	6.31E-03	-3.89E-01	6.52E-03
1	4	1.11E+00	4.97E-03	1.11E+00	5.32E-03	1.11E+00	5.46E-03
1	5	-9.91E-01	-3.06E-02	-9.87E-01	-3.25E-02	-9.83E-01	-2.78E-02
2	1	6.72E-01	3.19E-02	6.69E-01	3.21E-02	6.71E-01	3.18E-02
2	2	-2.12E+00	-3.16E-02	-2.13E+00	-3.16E-02	-2.12E+00	-3.15E-02
2	3	-1.32E+00	-8.22E-02	-1.31E+00	-8.56E-02	-1.32E+00	-8.19E-02
2	4	-1.26E+00	2.22E-02	-1.28E+00	2.23E-02	-1.25E+00	2.23E-02
2	5	-1.53E-01	-1.12E-02	-1.51E-01	-1.13E-02	-1.50E-01	-1.14E-02
3	1	2.49E-01	-1.62E-04	2.52E-01	-1.34E-04	2.49E-01	-2.06E-04
3	2	4.49E-02	1.17E-02	3.90E-02	1.30E-02	4.48E-02	1.16E-02
3	3	-1.18E+00	1.87E-02	-1.20E+00	1.85E-02	-1.18E+00	1.84E-02
3	4	1.67E+00	2.40E-02	1.66E+00	2.52E-02	1.67E+00	2.40E-02
3	5	-2.58E+00	-4.39E-02	-2.60E+00	-4.78E-02	-2.59E+00	-4.32E-02
4	1	1.78E+00	4.33E-02	1.78E+00	4.38E-02	1.78E+00	4.33E-02
4	2	-1.15E+00	-1.45E-01	-1.16E+00	-1.48E-01	-1.15E+00	-1.44E-01
4	3	1.85E+00	3.47E-02	1.84E+00	3.44E-02	1.85E+00	3.45E-02
4	4	1.03E+00	-4.01E-03	1.05E+00	-2.91E-03	1.03E+00	-4.52E-03
4	5	-2.21E-01	1.63E-02	-2.23E-01	1.75E-02	-2.22E-01	1.59E-02
5	1	2.91E-01	-6.68E-03	2.94E-01	-6.83E-03	2.90E-01	-6.79E-03
5	2	3.53E-01	2.70E-02	3.52E-01	2.93E-02	3.58E-01	2.72E-02
5	3	-1.84E+00	1.44E-02	-1.87E+00	1.38E-02	-1.84E+00	1.47E-02
5	4	1.72E+00	6.09E-02	1.69E+00	6.32E-02	1.71E+00	5.97E-02



# CONTINUATION OF TABLE XX

## EIGENVELOCITIES OF 12-BAR TRUSS

### 5-MODE CORRELATION

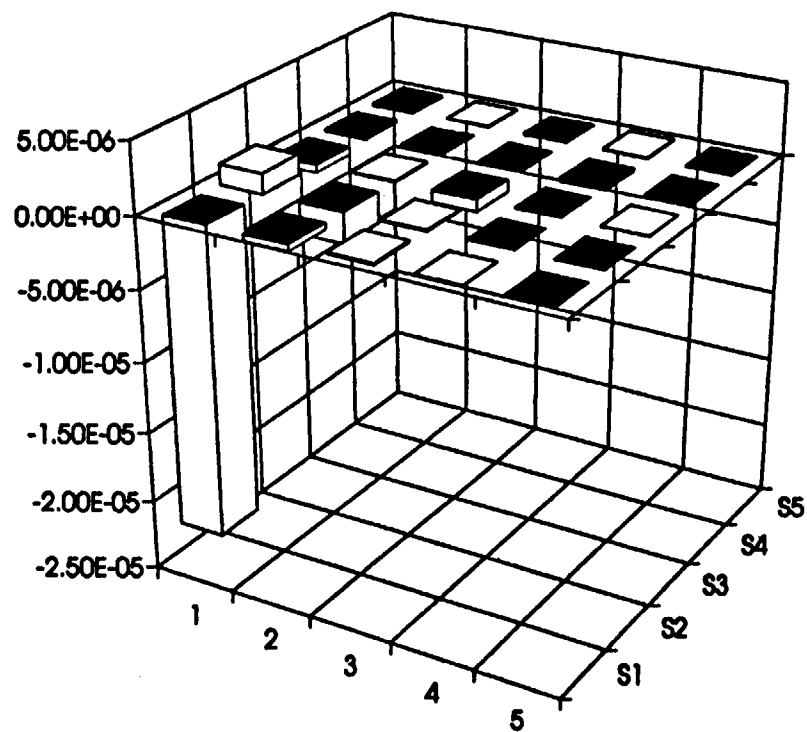
### METHOD 2, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	-1.78E+00	6.27E-02	-1.76E+00	6.46E-02	-1.76E+00	6.14E-02
6	1	2.95E+00	1.82E-03	2.96E+00	2.33E-03	2.94E+00	1.89E-03
6	2	1.95E+00	-1.06E-01	1.95E+00	-1.07E-01	1.95E+00	-1.06E-01
6	3	-1.25E+00	1.07E-01	-1.25E+00	1.10E-01	-1.25E+00	1.07E-01
6	4	-4.74E-01	3.06E-02	-4.82E-01	3.01E-02	-4.73E-01	3.09E-02
6	5	7.23E-01	3.77E-02	7.23E-01	3.78E-02	7.17E-01	3.80E-02
7	1	-1.18E-01	2.10E-03	-1.19E-01	2.09E-03	-1.18E-01	2.13E-03
7	2	-3.91E-01	1.66E-02	-3.97E-01	1.76E-02	-3.90E-01	1.65E-02
7	3	-5.23E-01	-8.73E-03	-5.35E-01	-9.49E-03	-5.22E-01	-8.58E-03
7	4	9.89E-01	1.26E-02	9.85E-01	1.27E-02	9.89E-01	1.28E-02
7	5	1.34E+00	1.05E-01	1.34E+00	1.08E-01	1.34E+00	1.08E-01
8	1	6.73E-01	3.19E-02	6.71E-01	3.21E-02	6.72E-01	3.18E-02
8	2	-2.17E+00	-3.50E-02	-2.18E+00	-3.48E-02	-2.17E+00	-3.51E-02
8	3	-1.40E+00	-9.08E-02	-1.38E+00	-9.42E-02	-1.40E+00	-9.09E-02
8	4	-1.48E+00	1.37E-02	-1.52E+00	1.40E-02	-1.49E+00	1.34E-02
8	5	-2.84E-01	-1.55E-02	-2.87E-01	-1.60E-02	-2.85E-01	-1.60E-02
9	1	-2.36E-01	4.54E-03	-2.38E-01	4.53E-03	-2.36E-01	4.57E-03
9	2	-7.65E-01	2.91E-02	-7.76E-01	3.08E-02	-7.65E-01	2.89E-02
9	3	-9.91E-01	-2.28E-02	-1.01E+00	-2.46E-02	-9.90E-01	-2.27E-02
9	4	1.68E+00	3.64E-02	1.67E+00	3.70E-02	1.68E+00	3.65E-02
9	5	1.44E+00	7.14E-02	1.42E+00	7.17E-02	1.42E+00	7.27E-02
10	1	1.78E+00	4.32E-02	1.78E+00	4.38E-02	1.77E+00	4.31E-02
10	2	-1.13E+00	-1.41E-01	-1.14E+00	-1.43E-01	-1.13E+00	-1.40E-01
10	3	1.75E+00	2.87E-02	1.74E+00	2.79E-02	1.74E+00	2.89E-02

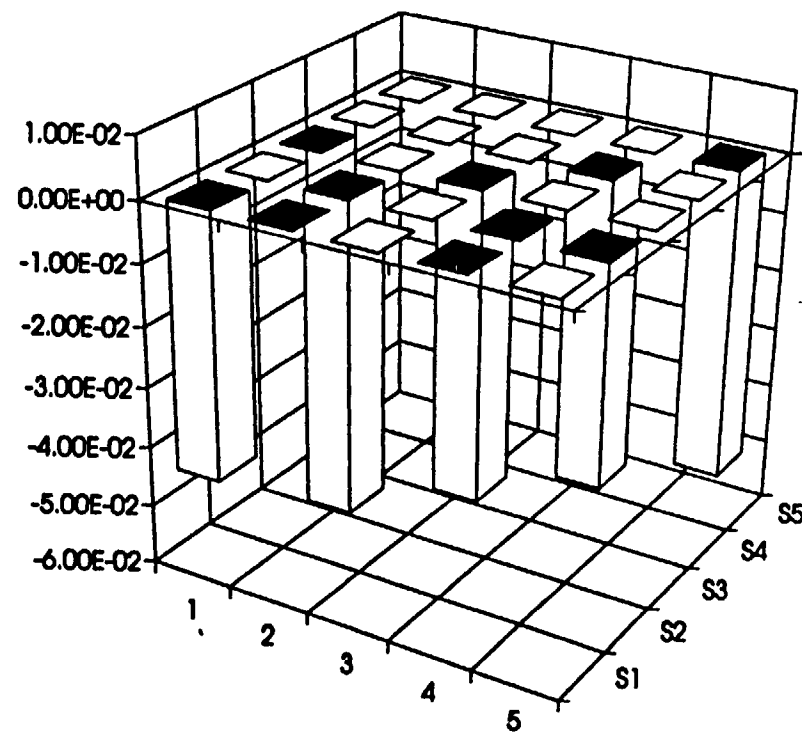
# CONTINUATION OF TABLE XX

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 2, 95 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	8.74E-01	-9.94E-03	8.90E-01	-9.74E-03	8.71E-01	-9.87E-03
10	5	-1.19E-01	6.78E-03	-1.18E-01	7.01E-03	-1.17E-01	6.64E-03
11	1	-2.36E-01	4.86E-03	-2.39E-01	4.83E-03	-2.35E-01	4.82E-03
11	2	-7.86E-01	2.52E-02	-7.98E-01	2.69E-02	-7.84E-01	2.48E-02
11	3	-1.06E+00	-3.59E-02	-1.08E+00	-3.79E-02	-1.06E+00	-3.53E-02
11	4	2.05E+00	9.49E-02	2.04E+00	9.58E-02	2.05E+00	9.55E-02
11	5	3.24E+00	-7.75E-02	3.28E+00	-8.14E-02	3.27E+00	-7.81E-02
12	1	2.95E+00	-6.02E-05	2.96E+00	5.13E-04	2.95E+00	-8.01E-05
12	2	1.98E+00	-1.01E-01	1.97E+00	-1.02E-01	1.97E+00	-1.01E-01
12	3	-1.30E+00	1.03E-01	-1.29E+00	1.08E-01	-1.29E+00	1.03E-01
12	4	-5.22E-01	2.79E-02	-5.32E-01	2.73E-02	-5.22E-01	2.81E-02
12	5	1.00E+00	2.95E-02	1.01E+00	2.98E-02	1.00E+00	2.96E-02

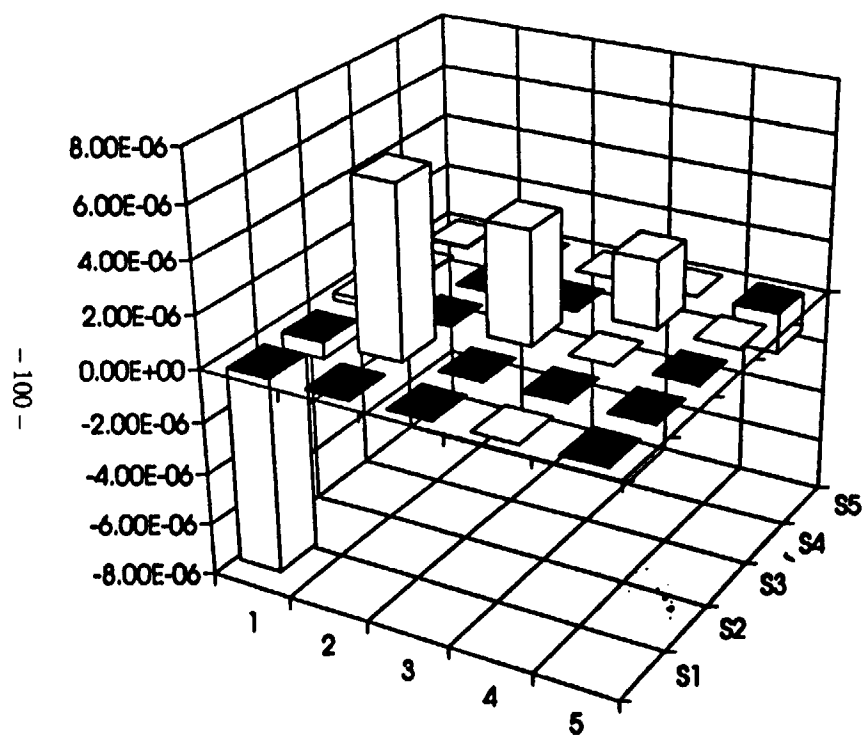


REAL

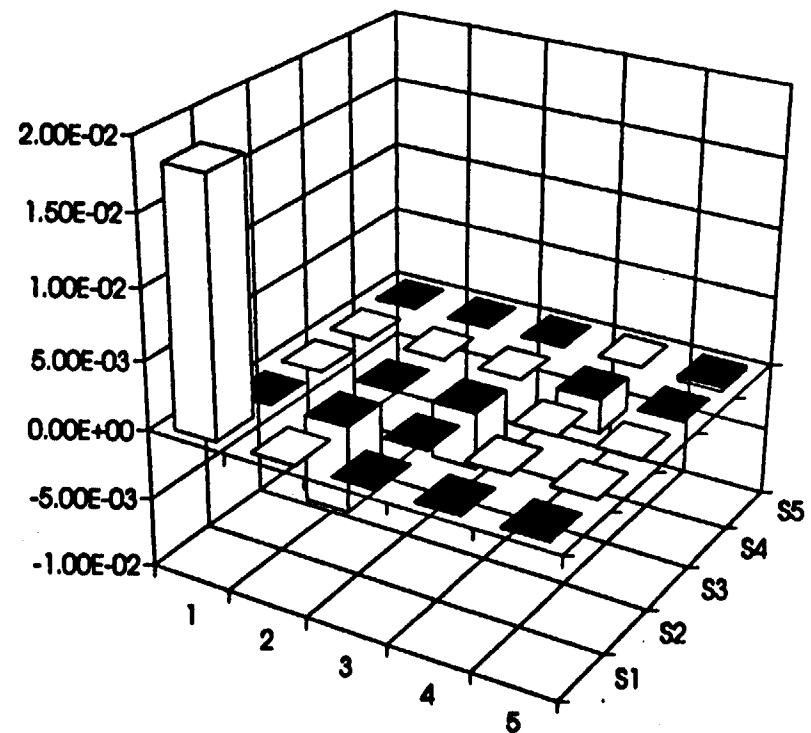


IMAGINARY

**Figure 27: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10),  
Correlated Modal Properties, and Test Spatial Properties  
5 Mode System Identification Of 95 Model Using Method 2**



REAL



IMAGINARY

**Figure 28: Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.11),  
Correlated Modal Properties, and Test Spatial Properties  
5 Mode System Identification Of 95 Model Using Method 2**

#### **4.6.3 Results for 12–BAR Truss, 90 Model, Method 1**

**1) Method 1, Two–Mode System Identification:** The results of this computer run are documented in tables XXI and XXII. Table XXI indicates that the improvements of the first and second eigenvalues are 100 percent. However, the imaginary portion of the first mode has worsened somewhat. Also, the eigenvelocities in table XVIII show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first and second modes. Overall, the results of the two–mode system identification appear quite satisfactory.

**2) Method 1, Five–Mode System Identification:** The results of this computer run are documented in tables XXIII and XXIV. Table XXIII indicates that there is a general improvement of the real portions of the second, third, and fourth eigenvalues. However, the magnitudes of the rest of the eigenvalues have worsened. Also, the eigenvelocities in table XVIII show some occasional improvement, but in general the eigenvelocities have worsened. Overall, the results of the five–mode system identification is unsatisfactory.

**TABLE XXI**

**EIGENVALUES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 1, 90 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.50E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.58E+02	-1.04E+03
2	-3.69E+02	-3.76E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE XXII**

**EIGENVELOCITIES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 1, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	6.99E-02	1.87E-03	7.07E-02	1.95E-03	7.04E-02	-1.58E-03
1	2	-1.28E-01	-8.50E-03	-1.41E-01	-9.50E-03	-1.38E-01	-8.53E-03
2	1	2.30E-01	1.08E-02	2.26E-01	1.08E-02	2.24E-01	-8.46E-03
2	2	-1.07E+00	-7.00E-02	-1.09E+00	-7.58E-02	-1.06E+00	-6.85E-02
3	1	8.40E-02	-4.70E-05	8.51E-02	-6.00E-05	8.50E-02	1.99E-04
3	2	2.53E-02	6.53E-03	1.96E-02	7.77E-03	1.98E-02	7.50E-03
4	1	6.05E-01	1.47E-02	6.02E-01	1.47E-02	6.00E-01	-1.27E-02
4	2	-5.74E-01	-1.01E-01	-5.91E-01	-1.08E-01	-5.76E-01	-1.02E-01
5	1	9.78E-02	-2.20E-03	9.93E-02	-2.32E-03	9.95E-02	2.23E-03
5	2	1.78E-01	2.15E-02	1.79E-01	2.49E-02	1.76E-01	2.34E-02
6	1	9.99E-01	6.60E-04	9.99E-01	6.14E-04	9.99E-01	-6.01E-04
6	2	9.87E-01	-2.99E-03	1.00E+00	0.00E+00	9.86E-01	-2.85E-03
7	1	-3.97E-02	7.12E-04	-4.03E-02	7.13E-04	-4.06E-02	-4.09E-04
7	2	-1.95E-01	-1.88E-03	-2.04E-01	-2.13E-03	-2.00E-01	-1.29E-03
8	1	2.30E-01	1.08E-02	2.26E-01	1.08E-02	2.24E-01	-8.42E-03
8	2	-1.09E+00	-7.30E-02	-1.11E+00	-7.89E-02	-1.09E+00	-7.13E-02
9	1	-7.92E-02	1.53E-03	-8.05E-02	1.54E-03	-8.10E-02	-9.55E-04
9	2	-3.81E-01	-5.32E-03	-3.98E-01	-6.00E-03	-3.91E-01	-4.29E-03
10	1	6.04E-01	1.46E-02	6.00E-01	1.47E-02	5.98E-01	-1.27E-02
10	2	-5.61E-01	-9.79E-02	-5.78E-01	-1.05E-01	-5.63E-01	-9.86E-02
11	1	-7.95E-02	1.64E-03	-8.07E-02	1.65E-03	-8.12E-02	-1.04E-03
11	2	-3.91E-01	-7.87E-03	-4.09E-01	-8.63E-03	-4.02E-01	-6.78E-03
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.00E+00	0.00E+00	1.01E+00	3.00E-03	1.00E+00	0.00E+00

**TABLE XXIII**

**EIGENVALUES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD1, 90 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.50E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	-3.25E+02	5.87E+03
4	-2.41E+02	-9.84E+03
5	-2.83E+02	1.72E+04

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	5.94E+03
4	-2.55E+02	-1.00E+04
5	-2.96E+02	1.76E+04

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-4.57E+02	-1.04E+03
2	-3.83E+02	3.90E+03
3	-3.32E+02	-6.02E+03
4	-2.49E+02	1.02E+04
5	-2.98E+02	-1.80E+04



**TABLE XXIV**

**EIGENVELOCITIES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 1, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	6.99E-02	1.87E-03	7.07E-02	1.95E-03	7.09E-02	-2.11E-03
1	2	-1.28E-01	-8.50E-03	-1.41E-01	-9.50E-03	1.31E-01	1.51E-03
1	3	-2.09E-01	-7.92E-03	2.16E-01	1.78E-03	2.26E-01	-5.54E-03
1	4	5.40E-01	-2.25E-02	5.43E-01	-2.29E-02	5.45E-01	2.60E-02
1	5	-3.11E-01	-1.61E-02	-3.01E-01	-1.74E-02	-3.01E-01	2.01E-02
2	1	2.30E-01	1.08E-02	2.26E-01	1.08E-02	2.24E-01	-8.62E-03
2	2	-1.07E+00	-7.00E-02	-1.09E+00	-7.58E-02	9.78E-01	1.17E-03
2	3	-7.23E-01	2.89E-02	7.00E-01	-5.09E-02	6.62E-01	4.80E-02
2	4	-5.98E-01	3.81E-02	-6.26E-01	4.03E-02	-6.57E-01	-4.30E-02
2	5	-4.80E-02	-4.59E-03	-4.60E-02	-4.59E-03	-4.64E-02	4.08E-03
3	1	8.40E-02	-4.70E-05	8.51E-02	-6.00E-05	8.62E-02	-1.09E-03
3	2	2.53E-02	6.53E-03	1.96E-02	7.77E-03	-1.21E-02	8.58E-03
3	3	-6.24E-01	-2.19E-02	6.42E-01	5.16E-03	6.51E-01	-7.95E-03
3	4	8.12E-01	-2.61E-02	8.10E-01	-2.56E-02	8.08E-01	3.10E-02
3	5	-7.97E-01	-3.08E-02	-7.92E-01	-3.42E-02	-7.92E-01	3.87E-02
4	1	6.05E-01	1.47E-02	6.02E-01	1.47E-02	6.00E-01	-1.25E-02
4	2	-5.74E-01	-1.01E-01	-5.91E-01	-1.08E-01	5.40E-01	-5.91E-02
4	3	1.00E+00	0.00E+00	-9.85E-01	2.56E-02	-9.46E-01	-3.49E-02
4	4	4.89E-01	-2.48E-02	5.13E-01	-2.55E-02	5.35E-01	2.76E-02
4	5	-6.84E-02	3.15E-03	-6.82E-02	3.64E-03	-6.86E-02	-3.23E-03
5	1	9.78E-02	-2.20E-03	9.93E-02	-2.32E-03	1.00E-01	1.04E-03
5	2	1.78E-01	2.15E-02	1.79E-01	2.49E-02	-1.55E-01	1.51E-02
5	3	-9.73E-01	-2.64E-02	1.00E+00	0.00E+00	1.00E+00	0.00E+00
5	4	8.45E-01	-1.02E-02	8.28E-01	-7.89E-03	8.21E-01	1.21E-02

# CONTINUATION OF TABLE XXIV

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	-5.62E-01	6.10E-03	-5.37E-01	6.36E-03	-5.37E-01	-6.04E-03
6	1	9.99E-01	6.60E-04	9.99E-01	6.14E-04	9.99E-01	-6.89E-04
6	2	9.87E-01	-2.99E-03	1.00E+00	0.00E+00	-8.93E-01	-5.30E-02
6	3	-6.77E-01	-6.83E-02	6.69E-01	5.41E-02	6.47E-01	-3.88E-02
6	4	-2.25E-01	2.55E-02	-2.35E-01	2.57E-02	-2.44E-01	-2.76E-02
6	5	2.26E-01	1.69E-02	2.20E-01	1.70E-02	2.22E-01	-1.78E-02
7	1	-3.97E-02	7.12E-04	-4.03E-02	7.13E-04	-4.07E-02	-2.14E-04
7	2	-1.95E-01	-1.88E-03	-2.04E-01	-2.13E-03	1.88E-01	1.12E-02
7	3	-2.75E-01	-8.61E-04	2.86E-01	-7.18E-03	2.92E-01	3.64E-03
7	4	4.82E-01	-1.60E-02	4.81E-01	-1.63E-02	4.81E-01	1.71E-02
7	5	4.15E-01	4.12E-02	4.09E-01	4.32E-02	4.08E-01	-4.81E-02
8	1	2.30E-01	1.08E-02	2.26E-01	1.08E-02	2.24E-01	-8.61E-03
8	2	-1.09E+00	-7.30E-02	-1.11E+00	-7.89E-02	1.00E+00	0.00E+00
8	3	-7.63E-01	3.28E-02	7.40E-01	-5.58E-02	6.98E-01	5.33E-02
8	4	-7.03E-01	3.87E-02	-7.40E-01	4.15E-02	-7.74E-01	-4.44E-02
8	5	-8.79E-02	-6.72E-03	-8.74E-02	-7.06E-03	-8.77E-02	6.73E-03
9	1	-7.92E-02	1.53E-03	-8.05E-02	1.54E-03	-8.10E-02	-8.99E-04
9	2	-3.81E-01	-5.32E-03	-3.98E-01	-6.00E-03	3.66E-01	2.01E-02
9	3	-5.22E-01	1.58E-03	5.41E-01	-1.71E-02	5.50E-01	1.08E-02
9	4	8.22E-01	-2.02E-02	8.15E-01	-2.01E-02	8.13E-01	2.18E-02
9	5	4.56E-01	3.27E-02	4.33E-01	3.26E-02	4.32E-01	-3.55E-02
10	1	6.04E-01	1.46E-02	6.00E-01	1.47E-02	5.99E-01	-1.25E-02
10	2	-5.61E-01	-9.79E-02	-5.78E-01	-1.05E-01	5.27E-01	-5.71E-02
10	3	9.47E-01	1.94E-03	-9.32E-01	2.18E-02	-8.97E-01	-3.04E-02

# CONTINUATION OF TABLE XXIV

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	4.16E-01	-2.40E-02	4.34E-01	-2.51E-02	4.53E-01	2.70E-02
10	5	-3.74E-02	1.18E-03	-3.60E-02	1.24E-03	-3.64E-02	-5.72E-04
11	1	-7.95E-02	1.64E-03	-8.07E-02	1.65E-03	-8.17E-02	-4.42E-04
11	2	-3.91E-01	-7.87E-03	-4.09E-01	-8.63E-03	3.79E-01	1.83E-02
11	3	-5.58E-01	7.80E-03	5.79E-01	-2.45E-02	5.91E-01	1.79E-02
11	4	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
11	5	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	1	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
12	2	1.00E+00	0.00E+00	1.01E+00	3.00E-03	-9.06E-01	-5.11E-02
12	3	-7.00E-01	-6.68E-02	6.92E-01	5.24E-02	6.69E-01	-3.67E-02
12	4	-2.48E-01	2.52E-02	-2.59E-01	2.55E-02	-2.69E-01	-2.76E-02
12	5	3.11E-01	1.63E-02	3.08E-01	1.67E-02	3.10E-01	-1.77E-02

#### **4.6.4 Results for 12-BAR Truss, 90 Model, Method 2**

**1) Method 2, Two-Mode System Identification:** The results of this computer run are documented in tables XXV and XXVI. Table XXV indicates that the improvements of the first and second eigenvalues are 100 percent. Also, the eigenvelocities in table XXVI show the same improvement, that is, nearly 100 percent improvements over the analytical results for the first and second modes. Overall, the results of the two-mode system identification appear quite satisfactory.

**2) Method 2, Five-Mode System Identification:** The results of this computer run are documented in tables XXVII and XXVIII. Table XXVII indicates that the improvements of the first, second, third, fourth, and fifth eigenvalues are near 100 percent. Also, the eigenvelocities in table XXVIII show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first, second, third, fourth and fifth modes. Overall, the results of the five-mode system identification appear quite satisfactory.

**TABLE XXV**

**EIGENVALUES OF 12-BAR TRUSS  
2-MODE CORRELATION  
METHOD 2, 90 MODEL**

<b>TEST EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.50E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

<b>ANALYSIS EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

<b>CORRELATED EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.50E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE XXVI**

**EIGENVELOCITIES OF 12-BAR TRUSS  
2-MODE CORREALTION  
METHOD 2, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	1.25E-01	7.66E-03	1.28E-01	8.47E-03	1.27E-01	7.95E-03
1	2	-1.54E-01	-7.83E-03	-1.69E-01	-8.56E-03	-1.66E-01	-8.33E-03
2	1	4.10E-01	3.35E-02	4.10E-01	3.53E-02	4.10E-01	3.37E-02
2	2	-1.29E+00	-6.41E-02	-1.30E+00	-6.91E-02	-1.28E+00	-6.46E-02
3	1	1.50E-01	5.10E-03	1.55E-01	5.82E-03	1.52E-01	5.29E-03
3	2	3.07E-02	7.39E-03	2.36E-02	8.91E-03	2.38E-02	7.43E-03
4	1	1.08E+00	6.36E-02	1.09E+00	6.86E-02	1.08E+00	6.37E-02
4	2	-6.94E-01	-1.10E-01	-7.09E-01	-1.18E-01	-6.95E-01	-1.10E-01
5	1	1.75E-01	2.11E-03	1.81E-01	2.69E-03	1.77E-01	2.20E-03
5	2	2.14E-01	2.26E-02	2.15E-01	2.62E-02	2.13E-01	2.30E-02
6	1	1.78E+00	6.29E-02	1.81E+00	7.06E-02	1.79E+00	6.32E-02
6	2	1.19E+00	-2.23E-02	1.20E+00	-1.99E-02	1.19E+00	-2.24E-02
7	1	-7.10E-02	-1.18E-03	-7.32E-02	-1.51E-03	-7.13E-02	-1.13E-03
7	2	-2.35E-01	1.42E-03	-2.44E-01	1.50E-03	-2.41E-01	1.32E-03
8	1	4.10E-01	3.36E-02	4.10E-01	3.54E-02	4.11E-01	3.37E-02
8	2	-1.32E+00	-6.73E-02	-1.33E+00	-7.23E-02	-1.31E+00	-6.78E-02
9	1	-1.42E-01	-2.16E-03	-1.46E-01	-2.80E-03	-1.42E-01	-2.05E-03
9	2	-4.59E-01	7.87E-04	-4.76E-01	7.31E-04	-4.71E-01	5.56E-04
10	1	1.08E+00	6.35E-02	1.09E+00	6.85E-02	1.08E+00	6.35E-02
10	2	-6.78E-01	-1.07E-01	-6.93E-01	-1.15E-01	-6.80E-01	-1.07E-01
11	1	-1.42E-01	-1.97E-03	-1.47E-01	-2.63E-03	-1.43E-01	-1.86E-03
11	2	-4.72E-01	-2.09E-03	-4.90E-01	-2.19E-03	-4.84E-01	-2.42E-03
12	1	1.79E+00	6.18E-02	1.82E+00	6.96E-02	1.79E+00	6.21E-02
12	2	1.21E+00	-1.89E-02	1.21E+00	-1.65E-02	1.20E+00	-1.90E-02

**TABLE XXVII**

**EIGENVALUES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 90 MODEL**

<b>TEST EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.50E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	-3.25E+02	5.87E+03
4	-2.41E+02	-9.84E+03
5	-2.83E+02	1.72E+04

<b>ANALYSIS EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.74E+02	1.02E+03
2	-3.86E+02	-3.83E+03
3	-3.42E+02	5.94E+03
4	-2.55E+02	-1.00E+04
5	-2.96E+02	1.76E+04

<b>CORRELATED EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-4.49E+02	1.02E+03
2	-3.69E+02	-3.76E+03
3	-3.25E+02	5.87E+03
4	-2.41E+02	-9.84E+03
5	-2.83E+02	1.72E+04

TABLE XXVIII

**EIGENVELOCITIES OF 12-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	2.05E-01	6.11E-03	2.09E-01	5.11E-03	2.06E-01	6.24E-03
1	2	-2.53E-01	-4.85E-03	-2.74E-01	-2.53E-03	-2.64E-01	-5.30E-03
1	3	-3.86E-01	-1.73E-02	4.01E-01	1.43E-02	-3.72E-01	-1.65E-02
1	4	1.11E+00	8.24E-03	1.11E+00	1.68E-03	1.11E+00	7.94E-03
1	5	-9.92E-01	-3.18E-02	-9.83E-01	-2.91E-02	-9.75E-01	-2.90E-02
2	1	6.72E-01	3.38E-02	6.66E-01	2.98E-02	6.70E-01	3.35E-02
2	2	-2.11E+00	-3.85E-02	-2.12E+00	-2.46E-02	-2.11E+00	-3.75E-02
2	3	-1.34E+00	4.41E-02	1.31E+00	-5.92E-02	-1.33E+00	4.36E-02
2	4	-1.23E+00	1.78E-02	-1.28E+00	2.64E-02	-1.22E+00	1.83E-02
2	5	-1.53E-01	-1.16E-02	-1.51E-01	-1.08E-02	-1.48E-01	-1.20E-02
3	1	2.46E-01	6.20E-04	2.51E-01	-9.53E-04	2.44E-01	5.02E-04
3	2	5.05E-02	1.05E-02	3.89E-02	1.29E-02	5.06E-02	1.06E-02
3	3	-1.15E+00	-4.85E-02	1.20E+00	4.23E-02	-1.15E+00	-4.81E-02
3	4	1.67E+00	2.82E-02	1.65E+00	1.97E-02	1.67E+00	2.72E-02
3	5	-2.54E+00	-4.82E-02	-2.59E+00	-3.91E-02	-2.57E+00	-4.82E-02
4	1	1.77E+00	4.84E-02	1.77E+00	3.78E-02	1.77E+00	4.79E-02
4	2	-1.14E+00	-1.45E-01	-1.16E+00	-1.44E-01	-1.14E+00	-1.45E-01
4	3	1.85E+00	1.28E-02	-1.84E+00	-2.33E-03	1.84E+00	1.24E-02
4	4	1.00E+00	-1.77E-03	1.05E+00	-6.31E-03	1.01E+00	-2.21E-03
4	5	-2.18E-01	1.43E-02	-2.22E-01	1.81E-02	-2.19E-01	1.51E-02
5	1	2.86E-01	-5.56E-03	2.93E-01	-7.76E-03	2.85E-01	-5.85E-03
5	2	3.52E-01	2.59E-02	3.50E-01	2.80E-02	3.63E-01	2.66E-02
5	3	-1.80E+00	-6.12E-02	1.86E+00	5.09E-02	-1.81E+00	-6.10E-02
5	4	1.73E+00	6.40E-02	1.69E+00	5.74E-02	1.72E+00	6.30E-02



# CONTINUATION OF TABLE XXVIII

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	-1.79E+00	5.47E-02	-1.75E+00	7.00E-02	-1.74E+00	5.71E-02
6	1	2.92E+00	1.09E-02	2.94E+00	-7.30E-03	2.92E+00	1.03E-02
6	2	1.94E+00	-9.77E-02	1.94E+00	-1.13E-01	1.94E+00	-9.78E-02
6	3	-1.25E+00	-1.35E-01	1.24E+00	1.35E-01	-1.24E+00	-1.34E-01
6	4	-4.64E-01	2.95E-02	-4.80E-01	3.15E-02	-4.61E-01	3.00E-02
6	5	7.21E-01	3.98E-02	7.20E-01	3.53E-02	7.07E-01	3.92E-02
7	1	-1.16E-01	1.73E-03	-1.19E-01	2.47E-03	-1.16E-01	1.83E-03
7	2	-3.84E-01	1.44E-02	-3.95E-01	1.88E-02	-3.82E-01	1.44E-02
7	3	-5.09E-01	-5.13E-03	5.33E-01	1.17E-03	-5.07E-01	-4.79E-03
7	4	9.89E-01	1.58E-02	9.81E-01	9.48E-03	9.89E-01	1.58E-02
7	5	1.33E+00	1.05E-01	1.34E+00	1.03E-01	1.33E+00	1.11E-01
8	1	6.73E-01	3.38E-02	6.68E-01	2.98E-02	6.72E-01	3.36E-02
8	2	-2.16E+00	-4.22E-02	-2.17E+00	-2.76E-02	-2.16E+00	-4.11E-02
8	3	-1.41E+00	5.07E-02	1.38E+00	-6.63E-02	-1.41E+00	5.04E-02
8	4	-1.44E+00	8.66E-03	-1.51E+00	1.88E-02	-1.45E+00	9.03E-03
8	5	-2.80E-01	-1.59E-02	-2.86E-01	-1.50E-02	-2.83E-01	-1.71E-02
9	1	-2.32E-01	3.78E-03	-2.37E-01	5.28E-03	-2.32E-01	3.89E-03
9	2	-7.51E-01	2.49E-02	-7.72E-01	3.32E-02	-7.51E-01	2.50E-02
9	3	-9.65E-01	-3.78E-03	1.01E+00	-4.40E-03	-9.63E-01	-3.08E-03
9	4	1.69E+00	4.12E-02	1.66E+00	3.14E-02	1.68E+00	4.11E-02
9	5	1.46E+00	7.57E-02	1.42E+00	6.67E-02	1.41E+00	7.76E-02
10	1	1.77E+00	4.83E-02	1.77E+00	3.78E-02	1.76E+00	4.77E-02
10	2	-1.12E+00	-1.41E-01	-1.13E+00	-1.39E-01	-1.11E+00	-1.41E-01
10	3	1.75E+00	1.57E-02	-1.74E+00	-6.84E-03	1.74E+00	1.53E-02

# CONTINUATION OF TABLE XXVIII

## EIGENVELOCITIES OF 12-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	8.55E-01	-7.30E-03	8.86E-01	-1.26E-02	8.48E-01	-7.78E-03
10	5	-1.19E-01	6.12E-03	-1.17E-01	7.37E-03	-1.15E-01	6.33E-03
11	1	-2.33E-01	4.09E-03	-2.38E-01	5.59E-03	-2.31E-01	4.10E-03
11	2	-7.72E-01	2.09E-02	-7.95E-01	2.94E-02	-7.66E-01	2.07E-02
11	3	-1.03E+00	7.26E-03	1.08E+00	-1.63E-02	-1.03E+00	8.19E-03
11	4	2.05E+00	1.01E-01	2.04E+00	8.87E-02	2.05E+00	1.01E-01
11	5	3.19E+00	-6.27E-02	3.26E+00	-9.17E-02	3.24E+00	-6.46E-02
12	1	2.93E+00	9.02E-03	2.95E+00	-9.12E-03	2.92E+00	8.36E-03
12	2	1.97E+00	-9.31E-02	1.97E+00	-1.08E-01	1.97E+00	-9.31E-02
12	3	-1.29E+00	-1.32E-01	1.29E+00	1.33E-01	-1.29E+00	-1.32E-01
12	4	-5.10E-01	2.68E-02	-5.29E-01	2.89E-02	-5.09E-01	2.73E-02
12	5	9.91E-01	3.24E-02	1.01E+00	2.64E-02	9.91E-01	3.18E-02

## 4.7 44-BAR Truss

The finite element model of the 44-BAR space truss is shown in figures 29 and 30. The MSC/NASTRAN finite element program was used to model the 44-BAR truss. As can be observed in figure 29, the 44-BAR space truss consists of three bays. Each bay of the truss consists of four longerons, four battens, and five diagonals. The numbers in figure 29 represent the finite element node numbers. Nodes 1 through 4 and 17 through 40 were grounded. The remaining nodes can translate in three orthogonal directions. We can see in figure 30 that there are 44 ROD elements (designated by the abbreviation RD). ROD elements in MSC/NASTRAN are general purpose elements with tension and compression capability. Therefore, the stiffness matrix of a ROD element is the same as the stiffness matrix of a simple spring. The mass of a ROD element is divided by two and the masses are placed on the respective two nodes surrounding the ROD element. Figure 30 also shows that three viscous dampers (VISC), designated by the abbreviations VS, are connected to every joint of the truss. The purpose of these elements is to resist the dynamic motion of the connecting joint in each translational direction. Thus, they represent the viscous damping in the joints of the truss. The VISC elements in the MSC/NASTRAN finite element program are general purpose elements that represent the general viscous dampers. In other words, the force produced by an element is equal to the velocity of the connecting degree-of-freedom times a damping coefficient.

ROD elements of the analytical 44-BAR truss finite element model are made of steel and have the following properties:

$$\text{Modulus of elasticity} = E = 30.0E6 \frac{\text{lb}}{\text{in}^2}$$

$$\text{Poisson's ratio} = \nu = .33$$

$$\text{Outer diameter of truss members} = O.D. = 1.0 \text{ in}$$

$$\text{Thickness of truss members} = t = 0.075 \text{ in}$$

$$\text{Mass density of truss members} = \rho = 0.0283 \frac{\text{lb} - \text{sec}^2}{\text{in}}$$

The VISC elements of the analytical 12-BAR truss finite element model have the following property:

$$\text{Damping coefficient} = C = 1.0 \frac{\text{lb} - \text{sec}}{\text{in}}$$

It should be noted that the selected value of the damping coefficient is quite large and based on this selection the real and imaginary parts of the eigenvalues and

eigenvectors have the same order of magnitude. This selection determines whether the methodology and the FORTRAN program in this study can support and handle structures with large damping.

In an effort to visualize the vibration motion of the analytical finite element model of the 44-BAR truss, the complex eigenvalue-eigenvector extraction of MSC/NASTRAN was activated, and the modal deformation plots have been included here. The modal deformation plots for the first four modes of the 44-BAR truss when it contains the analytical spatial properties are found in figures 31 through 38. Figures 31 through 34 represent the real portion of the modal deformations, while figures 35 through 38 represent the imaginary portion of the modal deformations. In order to better visualize and understand the motion of the 44-BAR truss, the modal deformations have been superimposed over the undeformed shape of the truss. We can see in figures 31 through 38 that the modal deformation plots represent the system level modes, where the entire structure is in motion. Mode 3 is almost completely torsional.

The following steps were taken to construct a test model :

- 1) The coefficients of the VISC elements 46, 48, 50 through 53, 55, 58, 60, and 62 were varied. When the damping coefficients of these elements were varied, nearly 40 percent of the elements of the damping matrix in the test model had different values than those in the analytical model.
- 2) The modulus of elasticity of the diagonal elements, namely ROD elements 29 through 37 were varied. When the modulus of elasticity was varied, the stiffness of the ROD elements would vary proportionally. In general, when the stiffness values of diagonal elements of a truss are varied, the bending stiffness is greatly influenced because a large portion of the bending stiffness and torsional stiffness is due to the action (tension and compression) in the diagonal elements.

The ROD elements and the VISC elements were varied so that the torsional and bending modes would couple. Therefore, the differences between the modal deformations of the test model and the analytical model are more severe than those of the 12-BAR truss. Variance of the properties of the diagonal ROD elements was not done randomly. When considering the diagonals, because of the connectivity of these elements to the joints, and the fact that they connect to the joint at an angle, the effective stiffness of a joint combined with that of a diagonal may be different from the effective stiffness of the joint combined with that of the longerons or of the battens. Selection of the VISC elements was done randomly by the author who assumed that the manufacturing process for the joints might have been different enough to create such different damping coefficients for the truss joints. However, in a practical structural system, the difference between the analytical spatial properties and the test spatial

properties cannot be as severe as 10 percent of 50 percent of the joints. The 5 percent or 10 percent reduction of the analytical spatial properties was used here only to examine the robustness of the methodology of this study.

In an effort to examine the accuracy and robustness of the methodology of the system identification and the FORTRAN program used for this study, the values, as described above were varied twice. The first variation of the values was a 5 percent reduction of the analytical model's properties. All the simulation computer runs for this 5 percent reduction were called the 95 model. The second variation of the values was a 10 percent reduction of the analytical model's properties. All of the simulation computer runs for this 10 percent reduction were called the 90 model. The results for the 95 model were satisfactory. Because the variation in spatial properties of the 90 model are more severe than those of the 95 model, only the results of the 90 model are presented here.

In an effort to visualize the vibration motion of the test finite element model of the 44-BAR truss, the complex eigenvalue-eigenvector extraction of the MSC/NASTRAN was again activated. The modal deformation plots have been included here in figures 39 through 46 for the first four modes of the 44-BAR truss, 90 model. When comparing the modal deformation plots of the 90 model with those of the analytical model, the only similarity that can be identified is between the real portion of the second and the third modes. Due to unsymmetric variance in the spatial properties of the analytical model it is expected to have such large differences between the modal deformations of the test model and those of the analytical model. We can see in figures 31 through 38 that the modal deformation plots represent the system level modes where the entire structure is in motion. Similar to the modal deformations of the analytical model, mode 3 is nearly pure torsional.

After the simulation computer runs were completed, the test modal data were saved on tape for the system identification process. Then the system identification was begun by exercising method 1 and method 2 presented in chapter III. The system identification of the 95 model (when using method 1) was performed with two modes (modes 1 and 3), three modes (modes 1, 3, and 5), four modes (modes 1, 3, 5, and 7), and five modes (modes 1, 3, 5, 7, and 9). The motivation for using a wide variety of modes in identifying the 90 model was to examine the degradation in the quality of the test eigenvalues and eigenvectors, if any. The results of the 90 model when using method 1 and method 2 have been documented and discussed for only two- and five-mode system identification to reduce the length of this document.

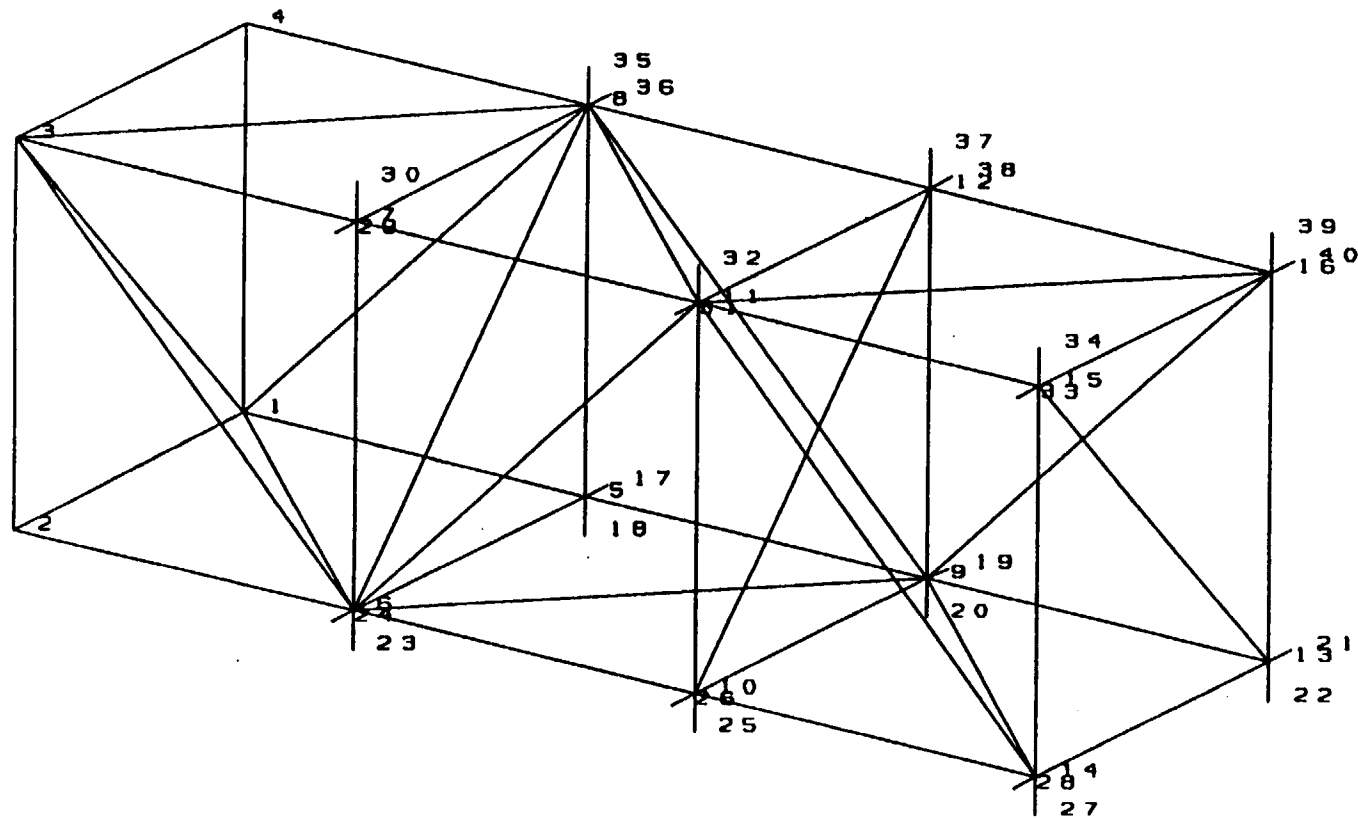


Figure 29: Analysis Finite Element Model Of 44-BAR Truss, Node Numbers

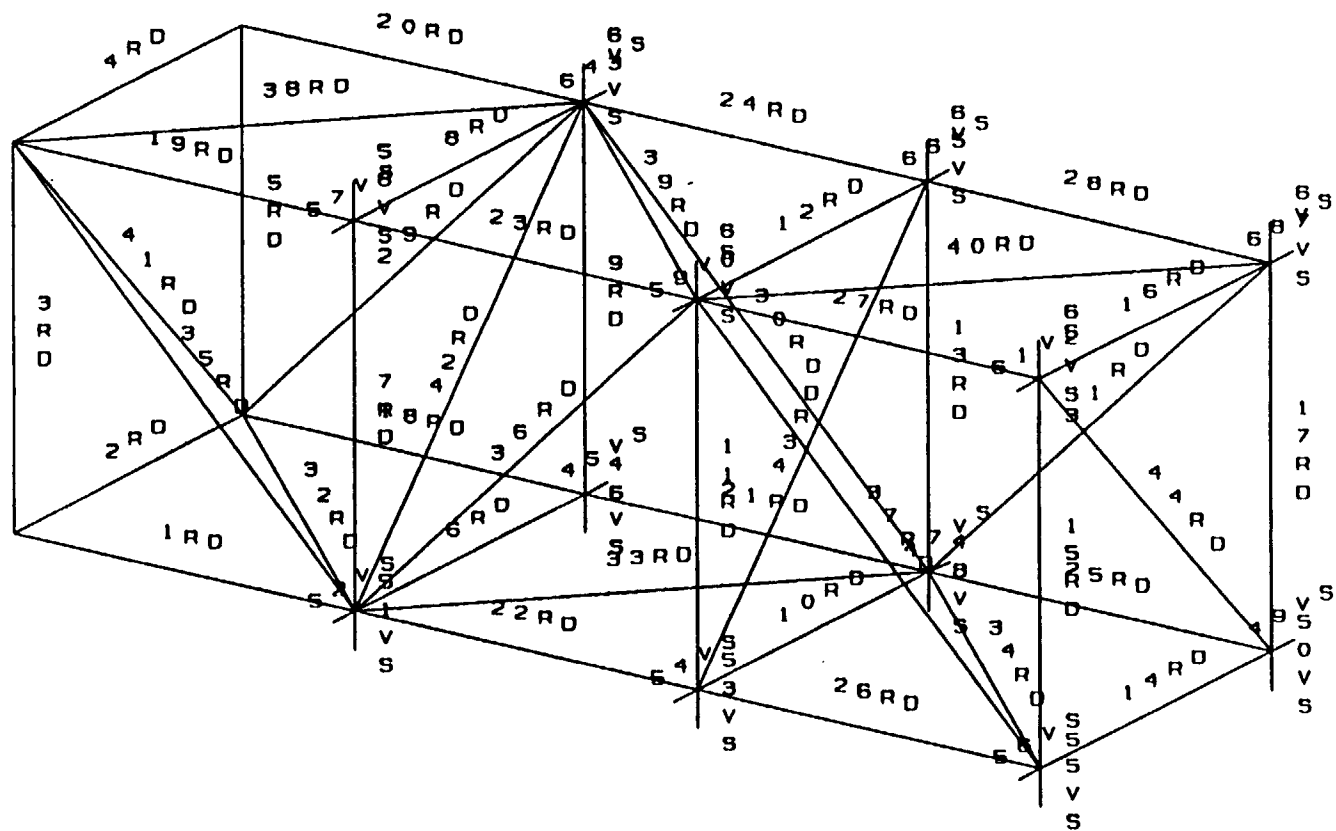
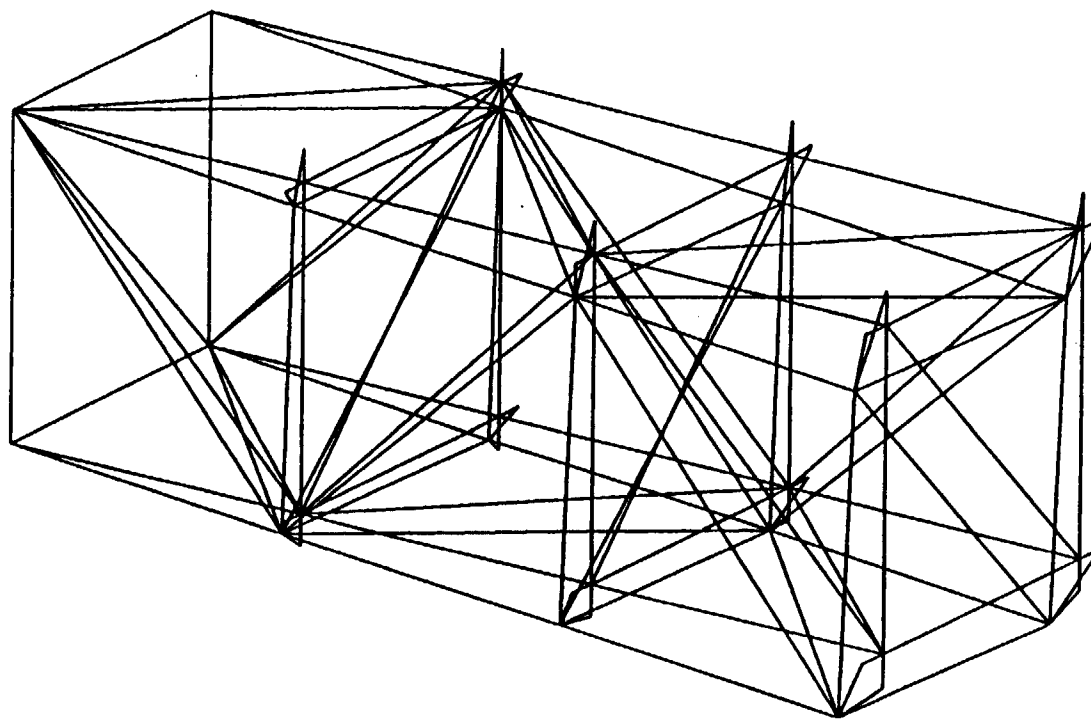
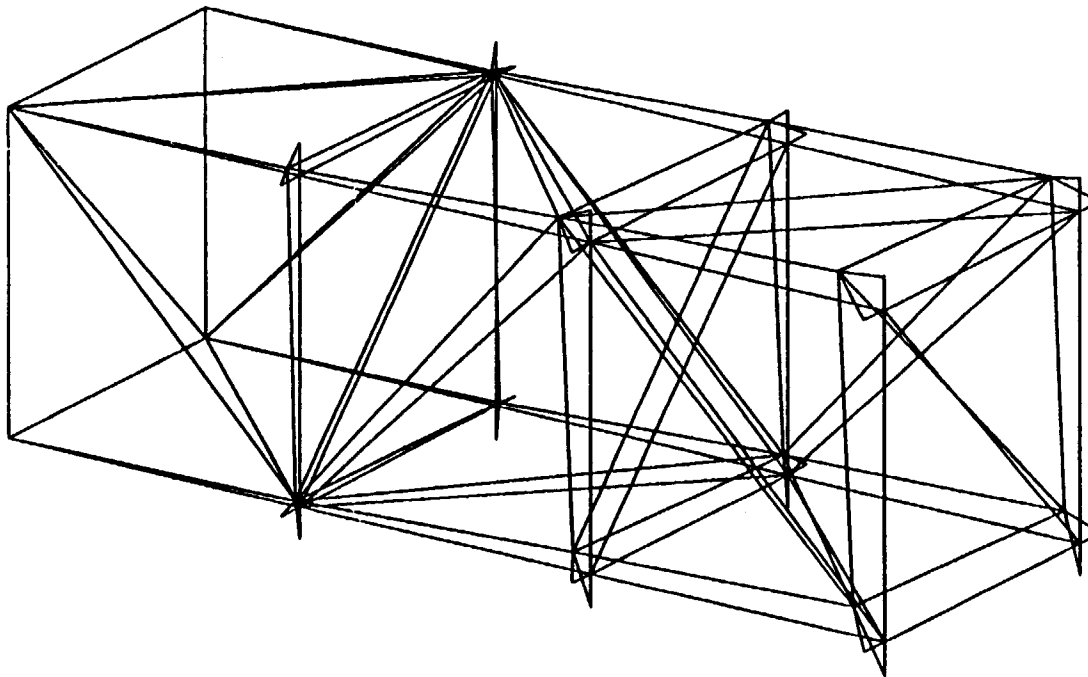


Figure 30: Analysis Finite Element Model Of 44-BAR Truss, Element Numbers

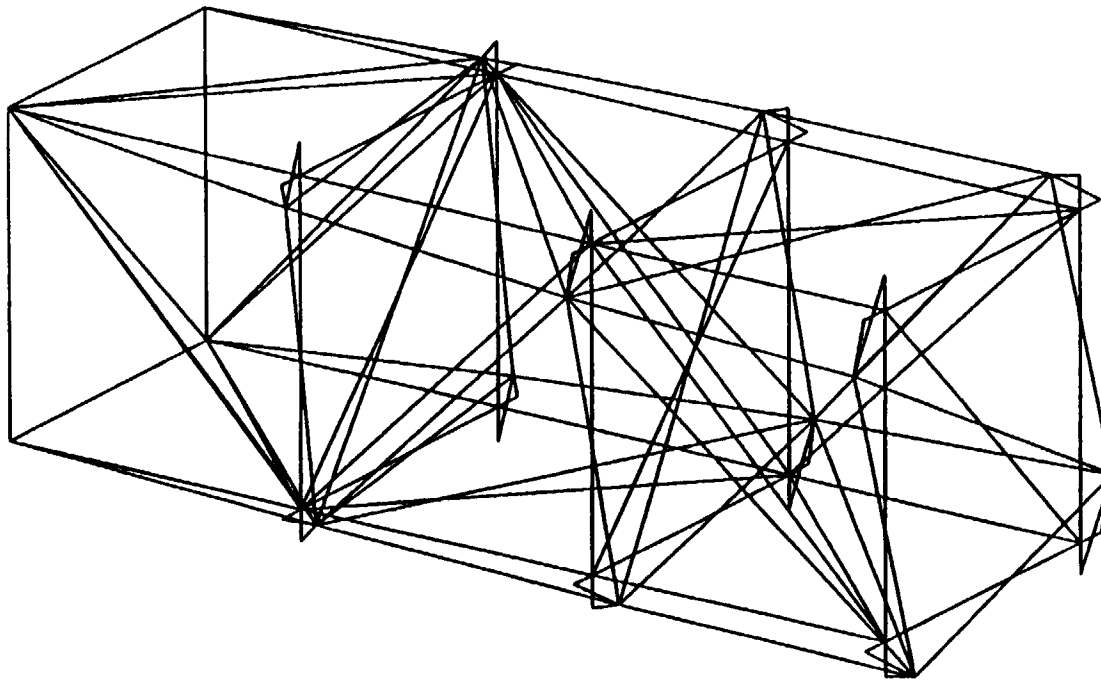


**Figure 31: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 1, Frequency=19.09284 Hz, Phase=0**

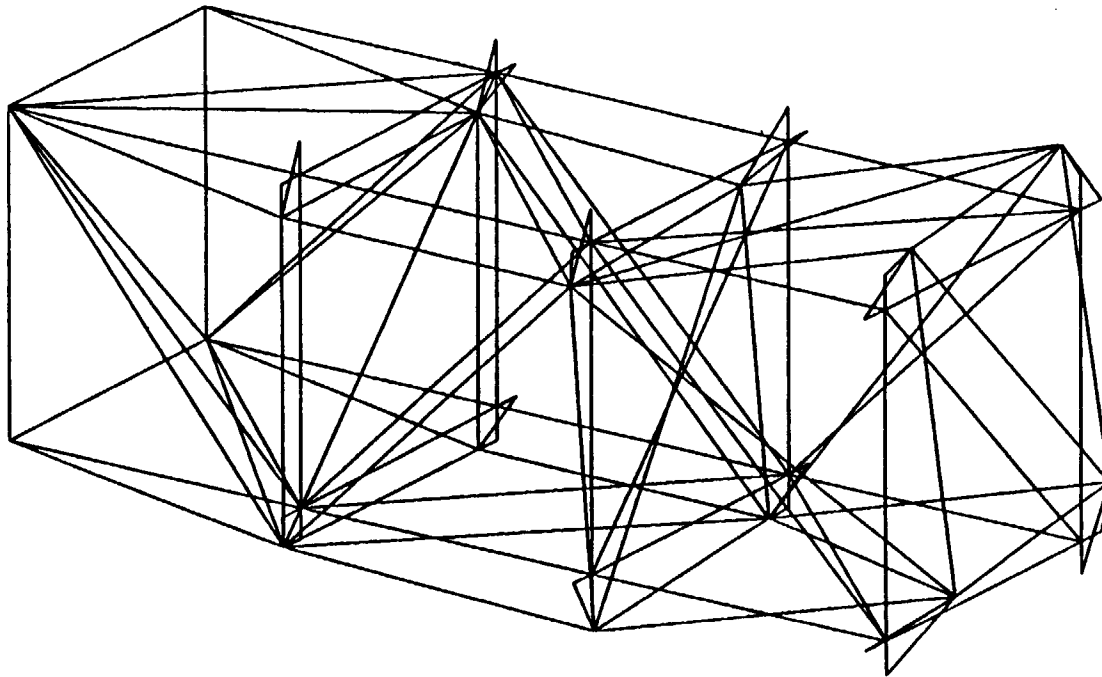




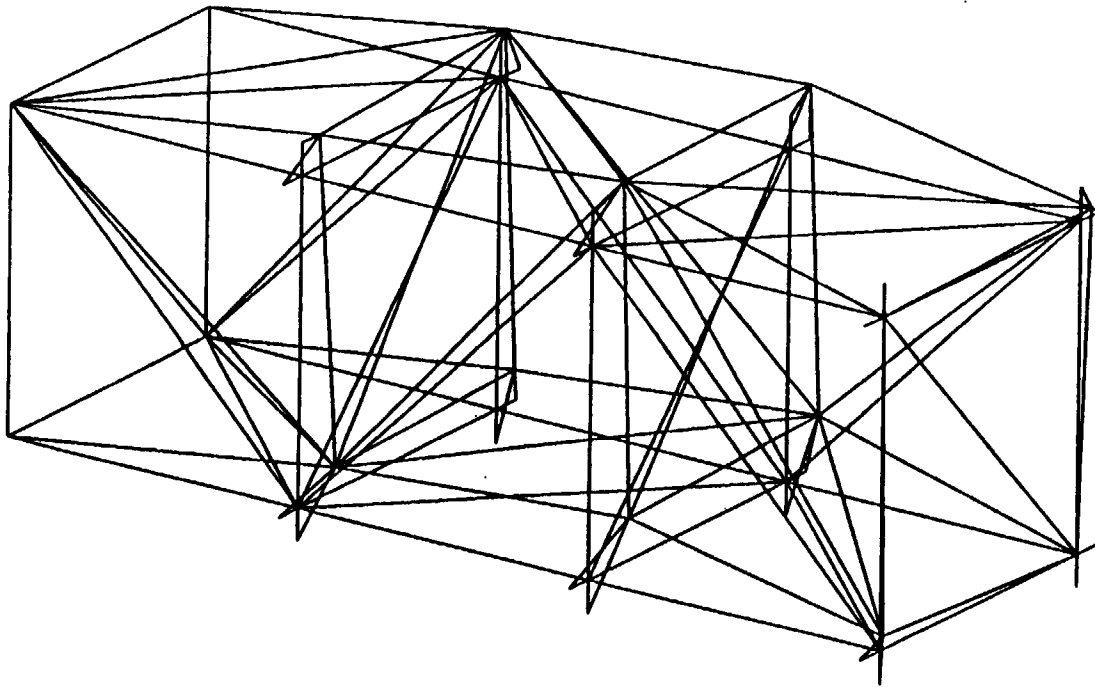
**Figure 32: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 2, Frequency=22.44332 Hz, Phase=0**



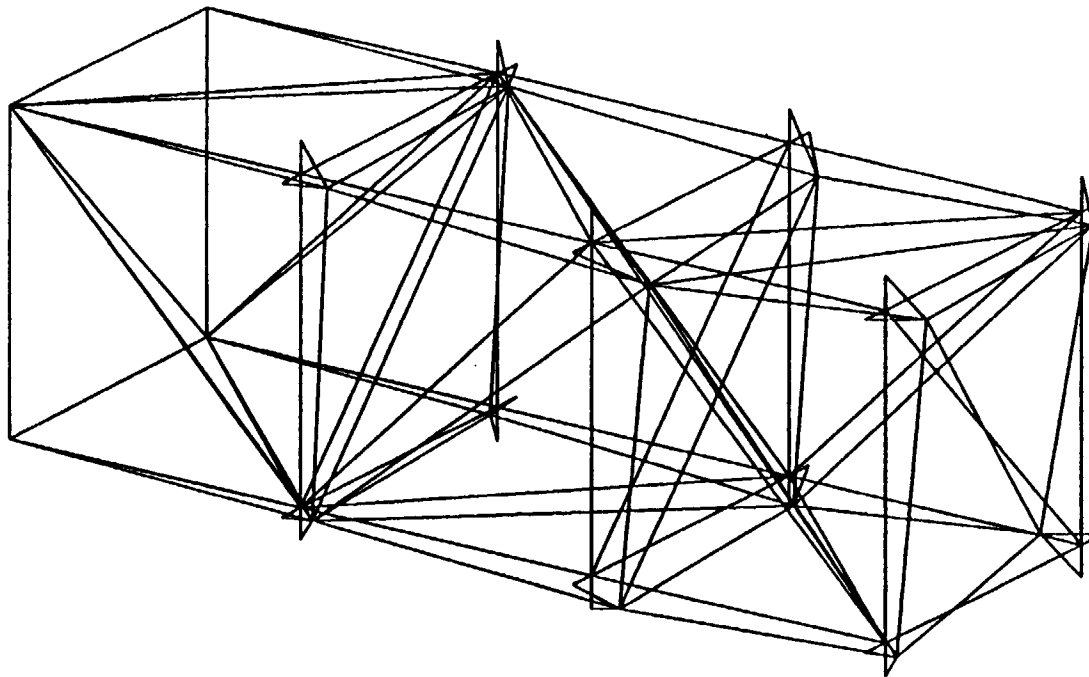
**Figure 33: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 3, Frequency=33.76233 Hz, Phase=0**



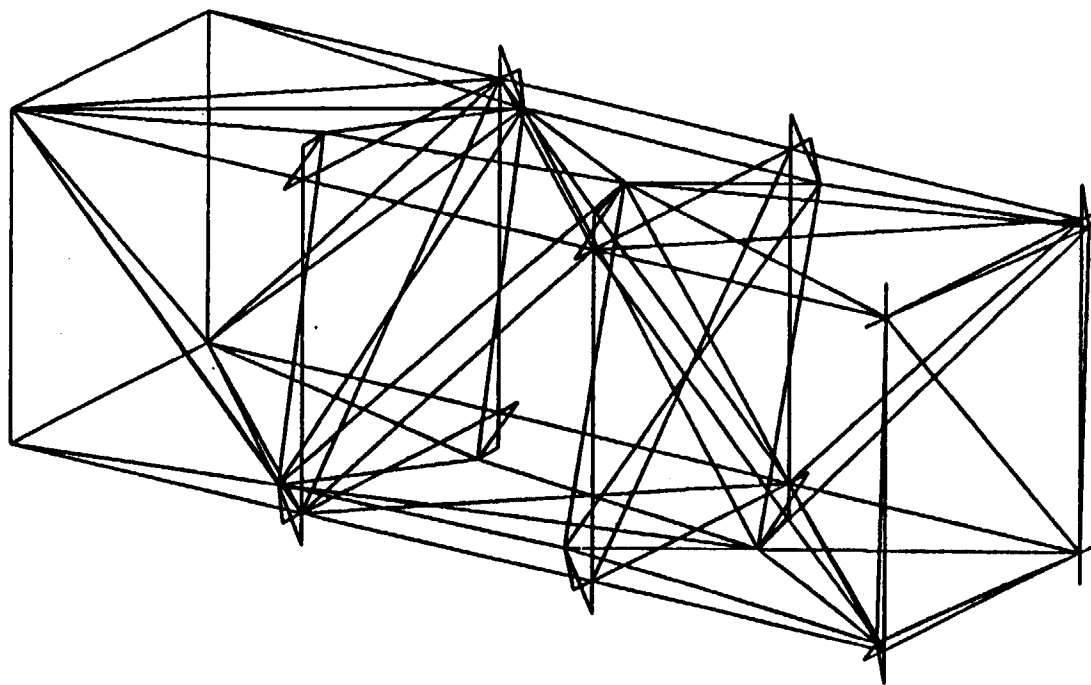
**Figure 34: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 4, Frequency=64.01659 Hz, Phase=0**



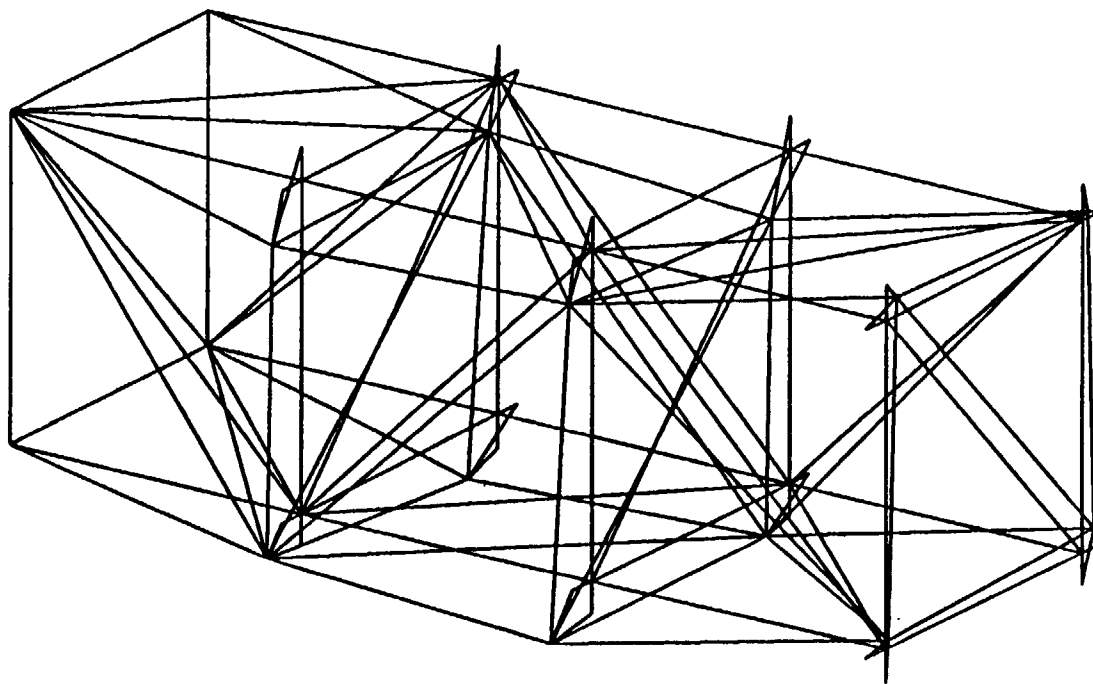
**Figure 35: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 1, Frequency=19.09284 Hz, Phase=-90**



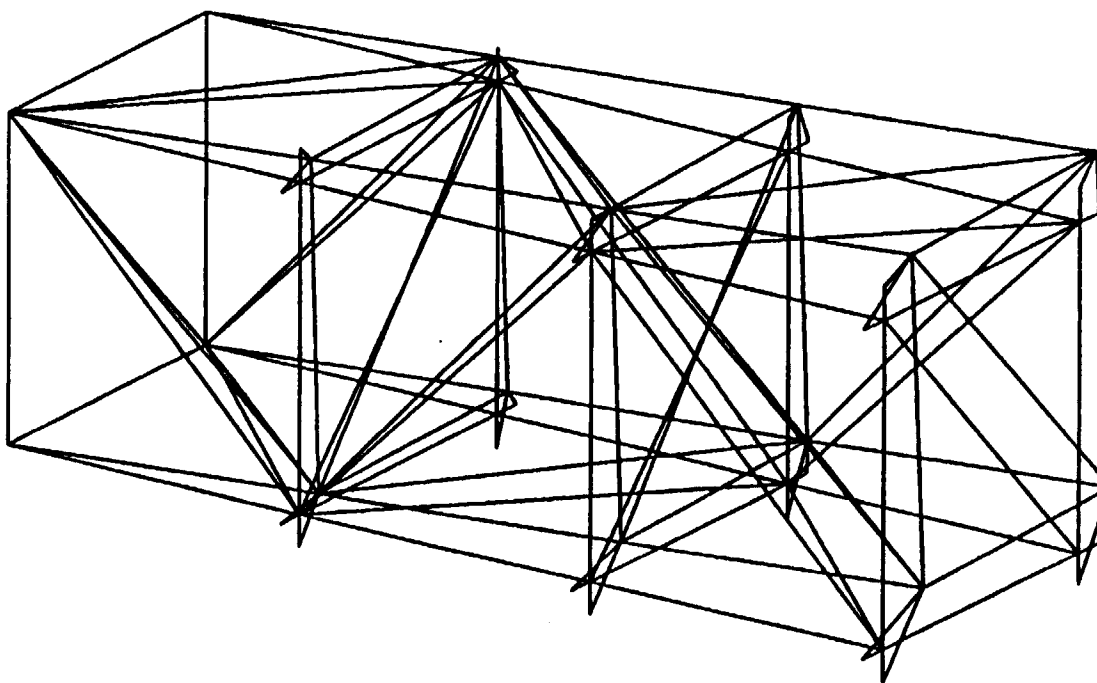
**Figure 36: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 2, Frequency=22.44332 Hz, Phase=-90**



**Figure 37: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 3, Frequency=33.76233 Hz, Phase=-90**

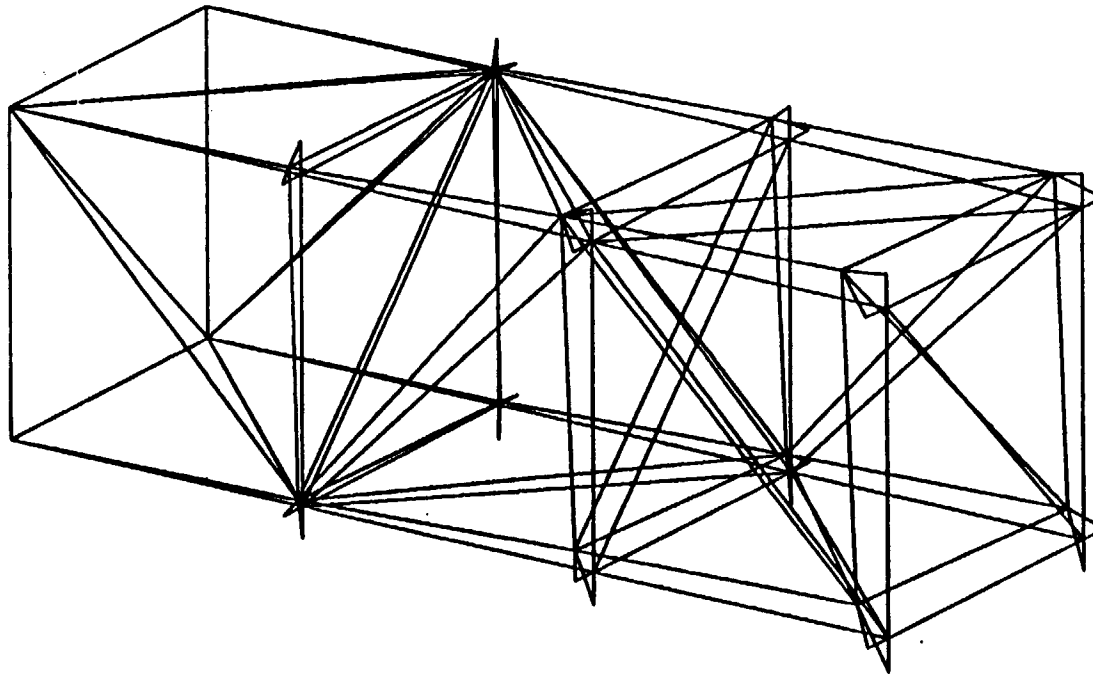


**Figure 38: Modal Deformation Plot Of Analysis Finite Element Model Of 44-BAR Truss,  
Mode 4, Frequency=64.01659 Hz, Phase=-90**

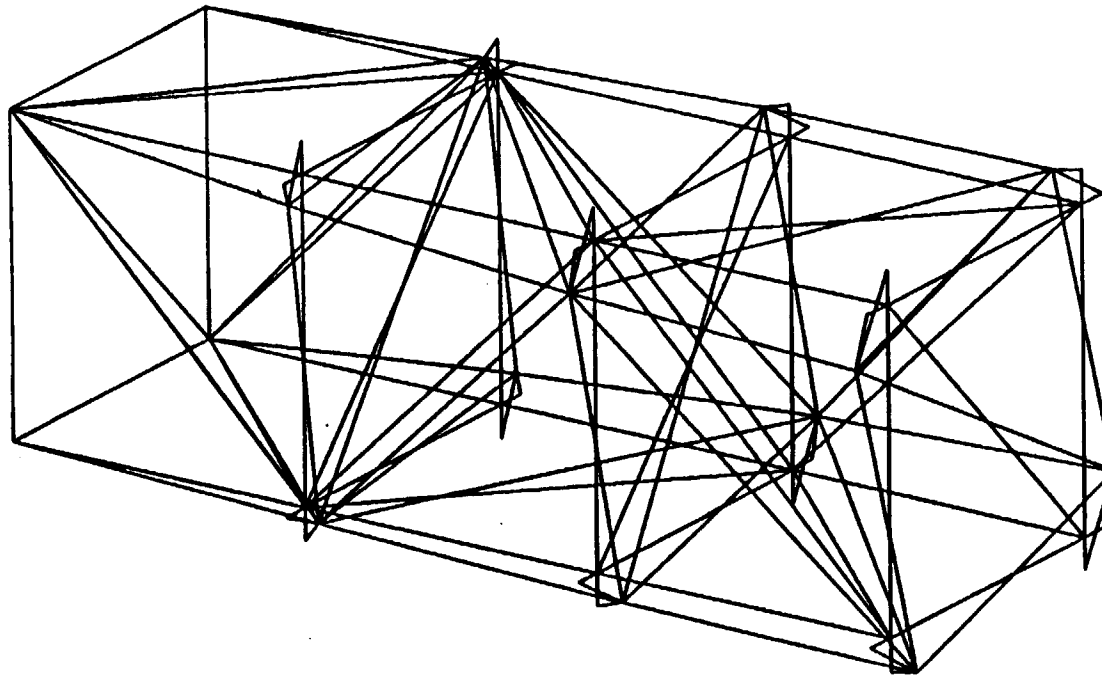


**Figure 39: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 1, Frequency=18.95565 Hz, Phase=0**

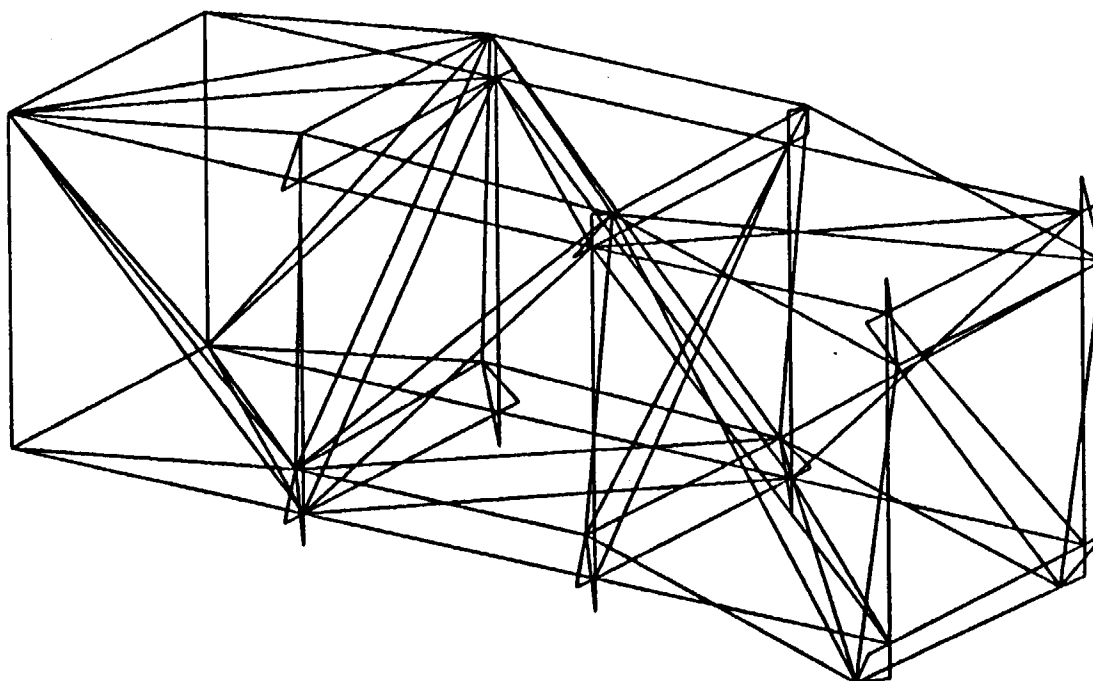




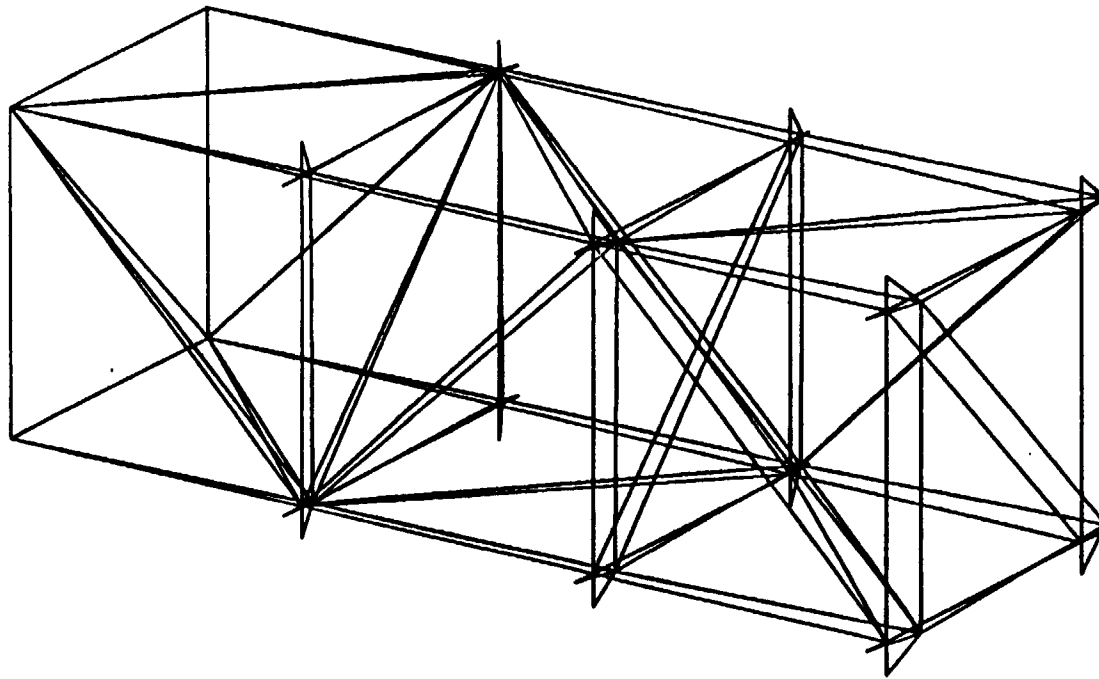
**Figure 40: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 2, Frequency=22.21710 Hz, Phase=0**



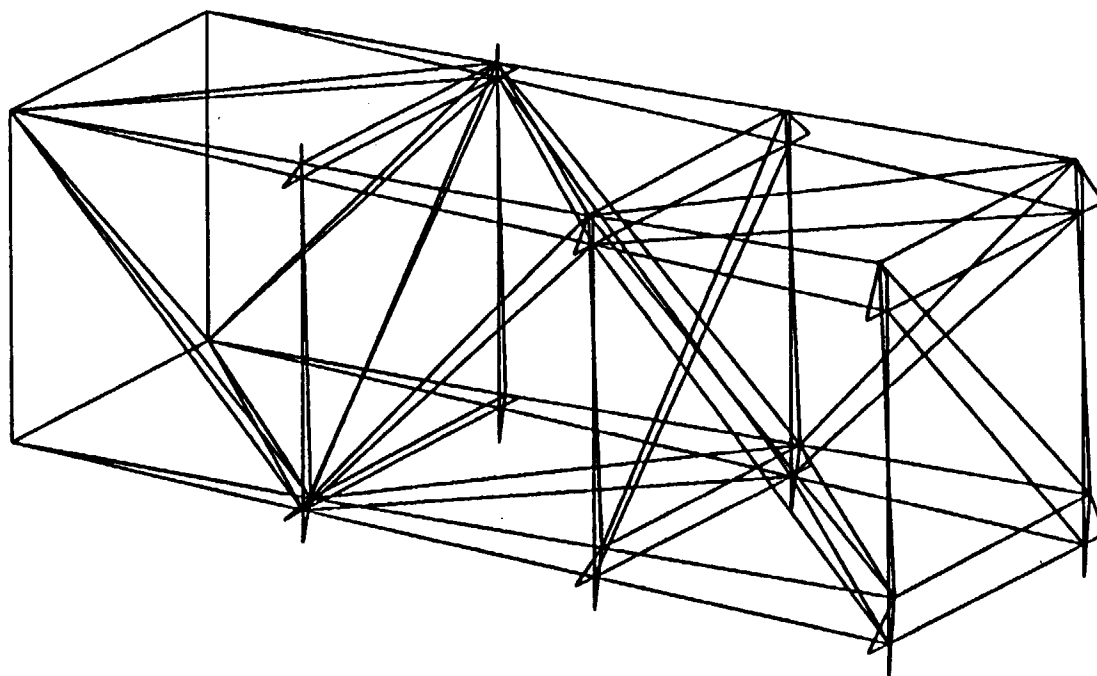
**Figure 41: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 3, Frequency=32.91091 Hz, Phase=0**



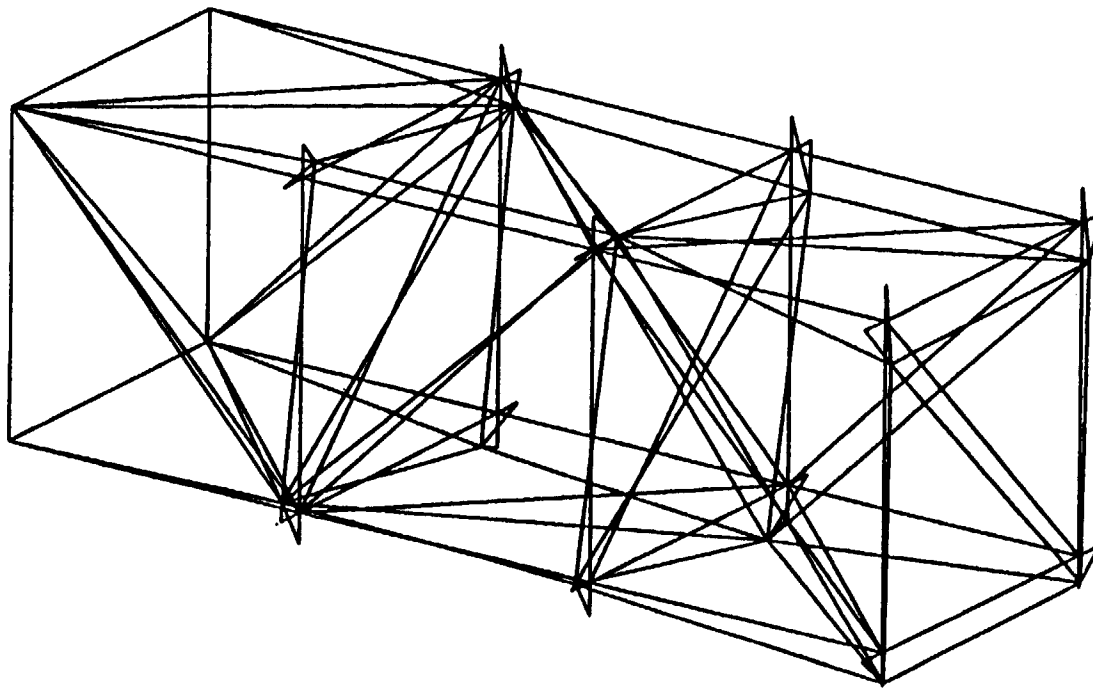
**Figure 42: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 4, Frequency=62.99915 Hz, Phase=0**



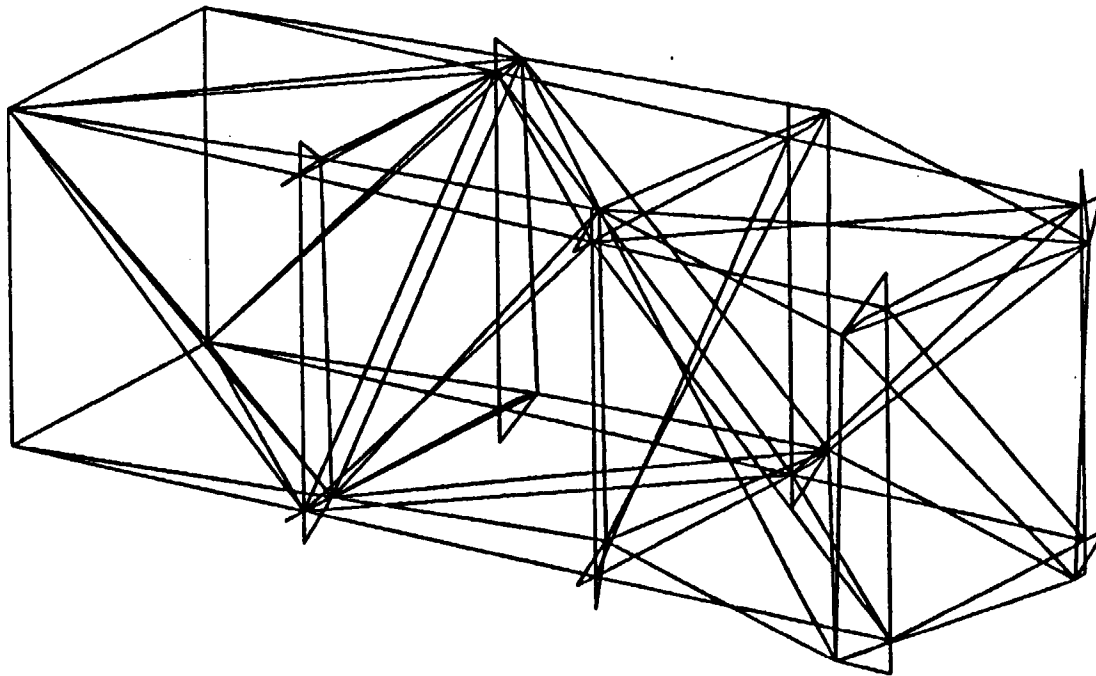
**Figure 43: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 1, Frequency=18.95565 Hz, Phase=-90**



**Figure 44: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 2, Frequency=22.21710 Hz, Phase=-90**



**Figure 45: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 3, Frequency=32.91091 Hz, Phase=-90**



**Figure 46: Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss,  
90 Model  
Mode 4, Frequency=62.99915 Hz, Phase=-90**

#### **4.7.1 Results for 44-BAR Truss, 90 Model, Method 1**

**1) Method 1, Two-Mode System Identification:** The results of this computer run are documented in tables XXIX and XXX. Table XXIX indicates that the improvement of the first and second eigenvalues are 100 percent. Also, the eigenvelocities in table XXX show the same improvement, that is, nearly 100 percent improvement over the analytical results for the first and second modes. Overall, the results of the two-mode system identification appear quite satisfactory.

**2) Method 1, Five-Mode System Identification:** The results of this computer run are documented in tables XXXI and XXXII. Table XXXI indicates that there is a general improvement of the real portions of the first, second, and fifth eigenvalues. However, the magnitudes of the rest of the eigenvalues have worsened. Also, the eigenvelocities in table XXXII show some occasional improvement, but in general the eigenvelocities have worsened. Overall, the results of the five-mode system identification are unsatisfactory.



**TABLE XXIX**

**EIGENVALUES OF 44-BAR TRUSS  
2-MODE CORRELATION  
METHOD 1, 90 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.87E+01	1.19E+02
2	-1.91E+01	-1.40E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.95E+01	-1.20E+02
2	-1.98E+01	1.41E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.87E+01	1.19E+02
2	-1.91E+01	-1.40E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE XXX**

**EIGENVELOCITIES OF 44-BAR TRUSS  
2-MODE CORRELATION  
METHOD 1, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	1.83E-03	5.09E-03	-1.78E-08	5.98E-09	2.67E-03	5.37E-03
1	2	1.55E-01	-6.13E-03	1.58E-01	1.10E-03	1.61E-01	-6.91E-03
2	1	-3.33E-01	1.41E-02	3.39E-01	-2.82E-03	-3.31E-01	1.39E-02
2	2	1.68E-01	-2.02E-02	1.53E-01	8.75E-04	1.59E-01	-1.90E-02
3	1	3.41E-01	-3.30E-03	-3.39E-01	2.82E-03	3.36E-01	-3.56E-03
3	2	1.50E-01	7.89E-03	1.53E-01	8.75E-04	1.50E-01	6.05E-03
4	1	2.00E-01	-4.73E-03	-2.09E-01	1.69E-03	2.05E-01	-5.43E-03
4	2	-2.79E-03	7.49E-03	3.57E-07	-9.82E-09	-2.75E-03	7.70E-03
5	1	-3.31E-01	1.36E-02	3.37E-01	-3.22E-03	-3.29E-01	1.34E-02
5	2	1.66E-01	-2.03E-02	1.51E-01	1.08E-03	1.58E-01	-1.91E-02
6	1	3.32E-01	-9.45E-04	-3.37E-01	3.22E-03	3.34E-01	-3.11E-03
6	2	1.58E-01	4.77E-03	1.51E-01	1.08E-03	1.49E-01	5.75E-03
7	1	-1.44E-03	-5.16E-03	3.54E-08	-5.68E-09	-2.67E-03	-5.37E-03
7	2	-1.56E-01	6.18E-03	-1.58E-01	-1.10E-03	-1.61E-01	6.91E-03
8	1	-3.26E-01	1.22E-02	3.39E-01	-2.82E-03	-3.31E-01	1.39E-02
8	2	1.57E-01	-1.73E-02	1.53E-01	8.75E-04	1.59E-01	-1.89E-02
9	1	3.34E-01	-1.28E-03	-3.39E-01	2.82E-03	3.36E-01	-3.53E-03
9	2	1.60E-01	5.00E-03	1.53E-01	8.75E-04	1.50E-01	5.98E-03
10	1	-2.02E-01	4.87E-03	2.09E-01	-1.69E-03	-2.05E-01	5.43E-03
10	2	5.16E-03	-7.68E-03	-3.84E-07	9.80E-09	2.75E-03	-7.70E-03
11	1	-3.23E-01	1.16E-02	3.37E-01	-3.22E-03	-3.29E-01	1.34E-02
11	2	1.56E-01	-1.74E-02	1.51E-01	1.08E-03	1.57E-01	-1.90E-02
12	1	3.38E-01	-2.95E-03	-3.37E-01	3.22E-03	3.34E-01	-3.14E-03
12	2	1.49E-01	7.65E-03	1.51E-01	1.08E-03	1.49E-01	5.82E-03

# CONTINUATION OF TABLE XXX

## EIGENVELOCITIES OF 44-BAR TRUSS 2-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
13	1	3.66E-03	1.01E-02	-4.24E-08	1.25E-08	5.32E-03	1.07E-02
13	2	3.09E-01	-1.18E-02	3.15E-01	1.76E-03	3.20E-01	-1.33E-02
14	1	-6.31E-01	3.74E-02	6.48E-01	-4.73E-03	-6.27E-01	3.77E-02
14	2	6.24E-01	-4.93E-02	6.02E-01	2.96E-03	6.18E-01	-4.89E-02
15	1	6.52E-01	2.85E-03	-6.48E-01	4.73E-03	6.47E-01	3.06E-03
15	2	5.91E-01	3.55E-03	6.02E-01	2.96E-03	6.01E-01	-1.11E-03
16	1	2.41E-01	-6.61E-03	-2.51E-01	1.06E-03	2.47E-01	-7.48E-03
16	2	-3.74E-03	8.93E-03	4.62E-07	-1.13E-08	-3.35E-03	9.25E-03
17	1	-6.37E-01	3.82E-02	6.54E-01	-4.49E-03	-6.33E-01	3.85E-02
17	2	6.32E-01	-4.96E-02	6.10E-01	2.73E-03	6.27E-01	-4.92E-02
18	1	6.46E-01	6.48E-03	-6.54E-01	4.49E-03	6.54E-01	2.94E-03
18	2	6.17E-01	-1.57E-03	6.10E-01	2.73E-03	6.09E-01	-1.06E-03
19	1	-2.89E-03	-1.03E-02	-1.79E-08	-1.12E-08	-5.32E-03	-1.07E-02
19	2	-3.10E-01	1.19E-02	-3.15E-01	-1.76E-03	-3.20E-01	1.33E-02
20	1	-6.19E-01	3.38E-02	6.48E-01	-4.73E-03	-6.27E-01	3.77E-02
20	2	6.05E-01	-4.42E-02	6.02E-01	2.96E-03	6.18E-01	-4.88E-02
21	1	6.40E-01	6.60E-03	-6.48E-01	4.73E-03	6.47E-01	3.11E-03
21	2	6.09E-01	-1.70E-03	6.02E-01	2.96E-03	6.01E-01	-1.23E-03
22	1	-2.43E-01	6.78E-03	2.51E-01	-1.06E-03	-2.47E-01	7.48E-03
22	2	6.07E-03	-9.13E-03	-4.57E-07	1.19E-08	3.35E-03	-9.25E-03
23	1	-6.25E-01	3.44E-02	6.54E-01	-4.49E-03	-6.33E-01	3.84E-02
23	2	6.13E-01	-4.43E-02	6.10E-01	2.73E-03	6.26E-01	-4.91E-02
24	1	6.59E-01	2.57E-03	-6.54E-01	4.49E-03	6.54E-01	2.88E-03
24	2	5.98E-01	3.89E-03	6.10E-01	2.73E-03	6.09E-01	-9.38E-04

# CONTINUATION OF TABLE XXX

## EIGENVELOCITIES OF 44-BAR TRUSS 2-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
25	1	3.66E-03	1.02E-02	1.27E-07	1.04E-08	5.35E-03	1.08E-02
25	2	3.12E-01	-1.28E-02	3.19E-01	2.75E-03	3.23E-01	-1.44E-02
26	1	-9.68E-01	6.64E-02	1.00E+00	0.00E+00	-9.66E-01	6.77E-02
26	2	1.03E+00	-7.20E-02	1.00E+00	1.25E-07	1.03E+00	-7.34E-02
27	1	1.00E+00	0.00E+00	-1.00E+00	6.34E-08	1.00E+00	-7.16E-05
27	2	9.77E-01	6.38E-03	1.00E+00	1.04E-07	1.00E+00	1.49E-04
28	1	2.79E-01	-9.14E-03	-2.91E-01	-3.58E-04	2.86E-01	-1.02E-02
28	2	-4.68E-03	1.02E-02	1.88E-07	-1.79E-08	-3.96E-03	1.07E-02
29	1	-9.61E-01	6.47E-02	9.92E-01	-1.01E-03	-9.58E-01	6.61E-02
29	2	1.02E+00	-7.24E-02	9.89E-01	1.13E-03	1.02E+00	-7.39E-02
30	1	9.77E-01	5.67E-03	-9.92E-01	1.01E-03	9.92E-01	8.97E-04
30	2	9.89E-01	-9.22E-04	9.89E-01	1.13E-03	9.89E-01	-1.01E-03
31	1	-2.88E-03	-1.04E-02	-3.71E-07	-9.98E-09	-5.35E-03	-1.08E-02
31	2	-3.13E-01	1.29E-02	-3.19E-01	-2.75E-03	-3.23E-01	1.44E-02
32	1	-9.53E-01	6.17E-02	1.00E+00	2.21E-08	-9.66E-01	6.76E-02
32	2	1.00E+00	-6.60E-02	1.00E+00	2.15E-07	1.03E+00	-7.33E-02
33	1	9.85E-01	4.90E-03	-1.00E+00	5.16E-08	1.00E+00	0.00E+00
33	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
34	1	-2.81E-01	9.36E-03	2.91E-01	3.58E-04	-2.86E-01	1.02E-02
34	2	6.93E-03	-1.05E-02	-3.02E-07	1.78E-08	3.96E-03	-1.07E-02
35	1	-9.46E-01	6.02E-02	9.92E-01	-1.01E-03	-9.58E-01	6.60E-02
35	2	9.93E-01	-6.66E-02	9.89E-01	1.13E-03	1.02E+00	-7.37E-02
36	1	9.92E-01	6.68E-04	-9.92E-01	1.01E-03	9.92E-01	8.27E-04
36	2	9.66E-01	5.62E-03	9.89E-01	1.13E-03	9.89E-01	-8.60E-04

**TABLE XXXI**

**EIGENVALUES OF 44-BAR TRUSS  
5-MODE CORREALTION  
METHOD 1, 90 MODEL**

<b>TEST EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-1.87E+01	1.19E+02
2	-1.91E+01	-1.40E+02
3	-1.85E+01	-2.07E+02
4	-1.47E+01	-3.96E+02
5	-1.56E+01	3.99E+02

<b>ANALYSIS EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-1.95E+01	-1.20E+02
2	-1.98E+01	1.41E+02
3	-1.93E+01	-2.12E+02
4	-1.51E+01	-4.06E+02
5	-1.62E+01	4.02E+02

<b>CORRELATED EIGENVALUES IN Hz</b>		
<b>MODE</b>	<b>REAL</b>	<b>IMAGINARY</b>
1	-1.91E+01	-1.21E+02
2	-1.95E+01	1.43E+02
3	-1.95E+01	2.18E+02
4	-1.56E+01	3.99E+02
5	-1.60E+01	-4.09E+02

TABLE XXXII

**EIGENVELOCITIES OF 44-BAR TRUSS  
5-MODE CORRELATION  
METHOD 1, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	1.83E-03	5.09E-03	-1.78E-08	5.98E-09	-4.24E-03	-6.17E-03
1	2	1.55E-01	-6.13E-03	1.58E-01	1.10E-03	1.55E-01	5.22E-03
1	3	5.32E-03	-1.03E-03	1.36E-09	1.96E-08	5.03E-03	2.37E-04
1	4	-1.89E-01	2.84E-02	-3.14E-01	7.86E-03	-2.00E-01	1.90E-02
1	5	-1.93E-01	1.82E-02	-1.55E-07	-1.05E-07	-2.34E-01	2.50E-02
2	1	-3.33E-01	1.41E-02	3.39E-01	-2.82E-03	-3.48E-01	-2.17E-02
2	2	1.68E-01	-2.02E-02	1.53E-01	8.75E-04	1.31E-01	1.06E-02
2	3	-4.62E-01	5.05E-03	-4.74E-01	5.42E-03	4.52E-01	2.77E-03
2	4	2.22E-01	-1.63E-01	1.00E+00	1.70E-06	9.94E-01	-5.30E-04
2	5	1.00E+00	0.00E+00	6.41E-01	-2.18E-02	1.00E+00	0.00E+00
3	1	3.41E-01	-3.30E-03	-3.39E-01	2.82E-03	3.34E-01	9.22E-03
3	2	1.50E-01	7.89E-03	1.53E-01	8.75E-04	1.53E-01	-6.19E-03
3	3	4.55E-01	-6.58E-03	4.74E-01	-5.42E-03	-4.65E-01	-5.94E-03
3	4	1.00E+00	5.68E-03	1.00E+00	-1.70E-08	2.72E-01	-1.51E-01
3	5	2.55E-01	-1.47E-01	-6.41E-01	2.18E-02	4.96E-01	-1.22E-01
4	1	2.00E-01	-4.73E-03	-2.09E-01	1.69E-03	2.13E-01	7.71E-03
4	2	-2.79E-03	7.49E-03	3.57E-07	-9.82E-09	3.80E-03	-4.89E-03
4	3	-2.42E-03	-6.44E-04	-3.92E-07	-1.17E-10	-1.69E-03	-1.07E-03
4	4	-1.86E-02	-2.29E-04	-8.90E-08	5.39E-08	1.08E-02	6.08E-03
4	5	1.79E-02	7.46E-03	2.01E-02	5.79E-03	8.03E-03	-6.79E-04
5	1	-3.31E-01	1.36E-02	3.37E-01	-3.22E-03	-3.45E-01	-2.10E-02
5	2	1.66E-01	-2.03E-02	1.51E-01	1.08E-03	1.30E-01	1.08E-02
5	3	-4.52E-01	6.02E-03	-4.63E-01	6.36E-03	4.43E-01	3.86E-03
5	4	2.04E-01	-1.52E-01	9.19E-01	-5.94E-03	9.14E-01	5.26E-03

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	9.22E-01	5.65E-03	5.90E-01	-1.66E-02	9.20E-01	-5.82E-03
6	1	3.32E-01	-9.45E-04	-3.37E-01	3.22E-03	3.39E-01	3.75E-03
6	2	1.58E-01	4.77E-03	1.51E-01	1.08E-03	1.41E-01	-5.78E-03
6	3	-4.56E-01	3.85E-03	-4.63E-01	6.36E-03	4.43E-01	1.56E-03
6	4	9.23E-01	-4.79E-03	9.19E-01	-5.94E-03	2.45E-01	-1.38E-01
6	5	2.55E-01	-1.35E-01	-5.90E-01	1.66E-02	4.59E-01	-1.15E-01
7	1	-1.44E-03	-5.16E-03	3.54E-08	-5.68E-09	4.24E-03	6.17E-03
7	2	-1.56E-01	6.18E-03	-1.58E-01	-1.10E-03	-1.55E-01	-5.22E-03
7	3	-6.50E-03	8.93E-04	-6.97E-09	-1.78E-08	-5.05E-03	-2.46E-04
7	4	1.90E-01	-2.97E-02	3.14E-01	-7.86E-03	2.00E-01	-1.89E-02
7	5	1.99E-01	-1.76E-02	2.79E-07	1.07E-07	2.34E-01	-2.49E-02
8	1	-3.26E-01	1.22E-02	3.39E-01	-2.82E-03	-3.55E-01	-1.66E-02
8	2	1.57E-01	-1.73E-02	1.53E-01	8.75E-04	1.41E-01	1.06E-02
8	3	4.55E-01	-3.36E-03	4.74E-01	-5.42E-03	-4.66E-01	-3.69E-03
8	4	2.20E-01	-1.57E-01	1.00E+00	1.69E-06	1.00E+00	0.00E+00
8	5	9.69E-01	1.09E-03	6.41E-01	-2.18E-02	9.96E-01	7.85E-05
9	1	3.34E-01	-1.28E-03	-3.39E-01	2.82E-03	3.41E-01	4.17E-03
9	2	1.60E-01	5.00E-03	1.53E-01	8.75E-04	1.42E-01	-6.20E-03
9	3	-4.65E-01	3.12E-03	-4.74E-01	5.42E-03	4.53E-01	5.20E-04
9	4	1.00E+00	0.00E+00	1.00E+00	0.00E+00	2.66E-01	-1.52E-01
9	5	2.76E-01	-1.48E-01	-6.41E-01	2.18E-02	5.01E-01	-1.22E-01
10	1	-2.02E-01	4.87E-03	2.09E-01	-1.69E-03	-2.13E-01	-7.71E-03
10	2	5.16E-03	-7.68E-03	-3.84E-07	9.80E-09	-3.82E-03	4.88E-03
10	3	9.16E-03	7.19E-04	3.46E-07	4.30E-10	1.68E-03	1.07E-03

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	2.16E-02	-2.02E-03	5.30E-08	-5.33E-08	-1.06E-02	-6.06E-03
10	5	-5.03E-03	-6.67E-03	-2.01E-02	-5.79E-03	-8.17E-03	6.93E-04
11	1	-3.23E-01	1.16E-02	3.37E-01	-3.22E-03	-3.52E-01	-1.60E-02
11	2	1.56E-01	-1.74E-02	1.51E-01	1.08E-03	1.40E-01	1.08E-02
11	3	4.46E-01	-4.35E-03	4.63E-01	-6.36E-03	-4.55E-01	-4.55E-03
11	4	2.02E-01	-1.46E-01	9.19E-01	-5.94E-03	9.20E-01	5.78E-03
11	5	8.94E-01	6.48E-03	5.90E-01	-1.66E-02	9.15E-01	-5.72E-03
12	1	3.38E-01	-2.95E-03	-3.37E-01	3.22E-03	3.32E-01	8.69E-03
12	2	1.49E-01	7.65E-03	1.51E-01	1.08E-03	1.51E-01	-5.76E-03
12	3	4.45E-01	-7.22E-03	4.63E-01	-6.36E-03	-4.54E-01	-6.85E-03
12	4	9.23E-01	4.54E-04	9.19E-01	-5.94E-03	2.51E-01	-1.37E-01
12	5	2.35E-01	-1.35E-01	-5.90E-01	1.66E-02	4.55E-01	-1.15E-01
13	1	3.66E-03	1.01E-02	-4.24E-08	1.25E-08	-8.47E-03	-1.23E-02
13	2	3.09E-01	-1.18E-02	3.15E-01	1.76E-03	3.09E-01	9.92E-03
13	3	1.05E-02	-2.01E-03	4.08E-09	3.96E-08	9.82E-03	4.20E-04
13	4	-3.64E-01	5.36E-02	-6.03E-01	1.32E-02	-3.84E-01	3.76E-02
13	5	-3.71E-01	3.62E-02	-3.13E-07	-1.99E-07	-4.49E-01	4.65E-02
14	1	-6.31E-01	3.74E-02	6.48E-01	-4.73E-03	-6.73E-01	-5.31E-02
14	2	6.24E-01	-4.93E-02	6.02E-01	2.96E-03	5.60E-01	3.10E-02
14	3	-7.82E-01	5.26E-03	-8.16E-01	7.97E-03	7.99E-01	5.25E-03
14	4	-1.38E-01	-1.16E-01	4.28E-01	3.23E-04	6.43E-01	4.89E-02
14	5	6.41E-01	4.62E-02	6.71E-01	-1.79E-02	5.88E-01	3.56E-02
15	1	6.52E-01	2.85E-03	-6.48E-01	4.73E-03	6.33E-01	5.50E-03
15	2	5.91E-01	3.55E-03	6.02E-01	2.96E-03	6.00E-01	-3.44E-04



# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
15	3	7.96E-01	-1.21E-02	8.16E-01	-7.97E-03	-7.88E-01	-8.17E-03
15	4	6.58E-01	4.93E-02	4.28E-01	3.21E-04	-1.06E-01	-1.14E-01
15	5	-1.07E-01	-1.11E-01	-6.71E-01	1.79E-02	5.49E-02	-8.81E-02
16	1	2.41E-01	-6.61E-03	-2.51E-01	1.06E-03	2.58E-01	9.90E-03
16	2	-3.74E-03	8.93E-03	4.62E-07	-1.13E-08	4.89E-03	-5.55E-03
16	3	-4.24E-03	-7.99E-04	-4.12E-07	-5.35E-09	-4.32E-03	-1.78E-03
16	4	-3.19E-01	-5.51E-02	-2.16E-06	8.15E-07	3.03E-01	7.71E-02
16	5	3.01E-01	7.47E-02	5.39E-01	5.52E-03	2.11E-01	4.18E-02
17	1	-6.37E-01	3.82E-02	6.54E-01	-4.49E-03	-6.79E-01	-5.42E-02
17	2	6.32E-01	-4.96E-02	6.10E-01	2.73E-03	5.67E-01	3.10E-02
17	3	-8.06E-01	4.95E-03	-8.42E-01	7.49E-03	8.22E-01	4.56E-03
17	4	-1.53E-01	-1.31E-01	4.81E-01	2.43E-03	7.22E-01	5.20E-02
17	5	7.15E-01	4.87E-02	7.51E-01	-2.27E-02	6.59E-01	4.25E-02
18	1	6.46E-01	6.48E-03	-6.54E-01	4.49E-03	6.52E-01	-3.05E-03
18	2	6.17E-01	-1.57E-03	6.10E-01	2.73E-03	5.90E-01	-6.76E-04
18	3	-8.14E-01	8.50E-04	-8.42E-01	7.49E-03	8.19E-01	-5.50E-04
18	4	7.29E-01	5.80E-02	4.81E-01	2.42E-03	-1.30E-01	-1.29E-01
18	5	-1.25E-01	-1.25E-01	-7.51E-01	2.27E-02	7.19E-02	-9.84E-02
19	1	-2.89E-03	-1.03E-02	-1.79E-08	-1.12E-08	8.47E-03	1.23E-02
19	2	-3.10E-01	1.19E-02	-3.15E-01	-1.76E-03	-3.09E-01	-9.93E-03
19	3	-1.29E-02	1.74E-03	-1.10E-07	-3.39E-08	-9.86E-03	-4.37E-04
19	4	3.65E-01	-5.60E-02	6.03E-01	-1.32E-02	3.84E-01	-3.75E-02
19	5	3.83E-01	-3.50E-02	4.22E-07	2.08E-07	4.49E-01	-4.64E-02
20	1	-6.19E-01	3.38E-02	6.48E-01	-4.73E-03	-6.85E-01	-4.44E-02

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
20	2	6.05E-01	-4.42E-02	6.02E-01	2.96E-03	5.77E-01	3.09E-02
20	3	8.01E-01	-6.41E-03	8.16E-01	-7.97E-03	-7.84E-01	-2.97E-03
20	4	-1.32E-01	-1.16E-01	4.28E-01	3.23E-04	6.54E-01	4.95E-02
20	5	6.36E-01	4.68E-02	6.71E-01	-1.79E-02	5.80E-01	3.57E-02
21	1	6.40E-01	6.60E-03	-6.48E-01	4.73E-03	6.45E-01	-3.17E-03
21	2	6.09E-01	-1.70E-03	6.02E-01	2.96E-03	5.82E-01	-3.20E-04
21	3	-7.91E-01	1.51E-03	-8.16E-01	7.97E-03	7.95E-01	7.27E-05
21	4	6.54E-01	5.00E-02	4.28E-01	3.21E-04	-1.17E-01	-1.15E-01
21	5	-1.13E-01	-1.12E-01	-6.71E-01	1.79E-02	6.36E-02	-8.82E-02
22	1	-2.43E-01	6.78E-03	2.51E-01	-1.06E-03	-2.58E-01	-9.90E-03
22	2	6.07E-03	-9.13E-03	-4.57E-07	1.19E-08	-4.92E-03	5.54E-03
22	3	1.08E-02	7.99E-04	2.28E-07	6.79E-09	4.29E-03	1.76E-03
22	4	3.23E-01	5.32E-02	2.22E-06	-8.21E-07	-3.03E-01	-7.71E-02
22	5	-2.89E-01	-7.36E-02	-5.39E-01	-5.52E-03	-2.11E-01	-4.17E-02
23	1	-6.25E-01	3.44E-02	6.54E-01	-4.49E-03	-6.92E-01	-4.52E-02
23	2	6.13E-01	-4.43E-02	6.10E-01	2.73E-03	5.85E-01	3.10E-02
23	3	8.25E-01	-6.12E-03	8.42E-01	-7.49E-03	-8.09E-01	-2.51E-03
23	4	-1.47E-01	-1.30E-01	4.81E-01	2.43E-03	7.34E-01	5.25E-02
23	5	7.14E-01	4.97E-02	7.51E-01	-2.27E-02	6.50E-01	4.26E-02
24	1	6.59E-01	2.57E-03	-6.54E-01	4.49E-03	6.39E-01	5.88E-03
24	2	5.98E-01	3.89E-03	6.10E-01	2.73E-03	6.08E-01	-7.10E-04
24	3	8.19E-01	-1.12E-02	8.42E-01	-7.49E-03	-8.13E-01	-7.63E-03
24	4	7.34E-01	5.84E-02	4.81E-01	2.42E-03	-1.19E-01	-1.28E-01

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS

### 5-MODE CORRELATION

### METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
24	5	-1.21E-01	-1.24E-01	-7.51E-01	2.27E-02	6.30E-02	-9.82E-02
25	1	3.66E-03	1.02E-02	1.27E-07	1.04E-08	-8.48E-03	-1.24E-02
25	2	3.12E-01	-1.28E-02	3.19E-01	2.75E-03	3.12E-01	1.11E-02
25	3	1.08E-02	-2.10E-03	1.45E-07	3.70E-08	1.04E-02	5.45E-04
25	4	-3.97E-01	6.12E-02	-6.61E-01	1.93E-02	-4.22E-01	3.82E-02
25	5	-4.06E-01	3.67E-02	-1.89E-07	-2.20E-07	-4.93E-01	5.46E-02
26	1	-9.68E-01	6.64E-02	1.00E+00	0.00E+00	-1.04E+00	-8.93E-02
26	2	1.03E+00	-7.20E-02	1.00E+00	1.25E-07	9.42E-01	4.61E-02
26	3	-9.39E-01	-3.53E-03	-1.00E+00	2.71E-07	1.00E+00	0.00E+00
26	4	2.31E-01	1.46E-01	-4.91E-01	9.88E-04	-8.30E-01	-8.21E-02
26	5	-8.37E-01	-7.98E-02	-8.97E-01	8.08E-03	-7.08E-01	-4.66E-02
27	1	1.00E+00	0.00E+00	-1.00E+00	6.34E-08	9.85E-01	1.05E-02
27	2	9.77E-01	6.38E-03	1.00E+00	1.04E-07	1.00E+00	0.00E+00
27	3	9.76E-01	-7.71E-03	1.00E+00	-6.09E-08	-9.62E-01	-1.80E-03
27	4	-8.48E-01	-6.72E-02	-4.91E-01	9.91E-04	2.00E-01	1.56E-01
27	5	2.14E-01	1.53E-01	8.97E-01	-8.08E-03	-2.05E-02	1.06E-01
28	1	2.79E-01	-9.14E-03	-2.91E-01	-3.58E-04	3.00E-01	1.28E-02
28	2	-4.68E-03	1.02E-02	1.88E-07	-1.79E-08	5.87E-03	-6.22E-03
28	3	-5.93E-03	-9.52E-04	1.31E-08	-1.06E-08	-6.67E-03	-2.38E-03
28	4	-5.87E-01	-1.07E-01	-4.01E-06	1.47E-06	5.64E-01	1.37E-01
28	5	5.53E-01	1.32E-01	1.00E+00	0.00E+00	3.91E-01	8.16E-02
29	1	-9.61E-01	6.47E-02	9.92E-01	-1.01E-03	-1.04E+00	-8.74E-02
29	2	1.02E+00	-7.24E-02	9.89E-01	1.13E-03	9.31E-01	4.67E-02
29	3	-9.18E-01	-2.09E-03	-9.75E-01	1.83E-03	9.75E-01	1.86E-03

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS

### 5-MODE CORRELATION

### METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
29	4	2.12E-01	1.32E-01	-4.47E-01	3.51E-03	-7.57E-01	-7.91E-02
29	5	-7.65E-01	-7.71E-02	-8.19E-01	3.09E-03	-6.45E-01	-3.90E-02
30	1	9.77E-01	5.67E-03	-9.92E-01	1.01E-03	9.92E-01	-7.74E-04
30	2	9.89E-01	-9.22E-04	9.89E-01	1.13E-03	9.68E-01	1.07E-03
30	3	-9.40E-01	-7.96E-03	-9.75E-01	1.83E-03	9.53E-01	-9.32E-03
30	4	-7.79E-01	-4.74E-02	-4.47E-01	3.51E-03	1.68E-01	1.43E-01
30	5	1.52E-01	1.39E-01	8.19E-01	-3.09E-03	-6.70E-03	9.72E-02
31	1	-2.88E-03	-1.04E-02	-3.71E-07	-9.98E-09	8.48E-03	1.24E-02
31	2	-3.13E-01	1.29E-02	-3.19E-01	-2.75E-03	-3.12E-01	-1.11E-02
31	3	-1.32E-02	1.84E-03	3.55E-08	-3.17E-08	-1.04E-02	-5.65E-04
31	4	3.98E-01	-6.39E-02	6.61E-01	-1.93E-02	4.22E-01	-3.82E-02
31	5	4.19E-01	-3.52E-02	6.40E-07	2.31E-07	4.92E-01	-5.46E-02
32	1	-9.53E-01	6.17E-02	1.00E+00	2.21E-08	-1.06E+00	-7.88E-02
32	2	1.00E+00	-6.60E-02	1.00E+00	2.15E-07	9.63E-01	4.60E-02
32	3	1.00E+00	0.00E+00	1.00E+00	0.00E+00	-9.39E-01	9.55E-03
32	4	2.34E-01	1.35E-01	-4.91E-01	9.89E-04	-8.15E-01	-8.20E-02
32	5	-7.85E-01	-7.81E-02	-8.97E-01	8.08E-03	-7.20E-01	-4.67E-02
33	1	9.85E-01	4.90E-03	-1.00E+00	5.16E-08	1.00E+00	0.00E+00
33	2	1.00E+00	0.00E+00	1.00E+00	0.00E+00	9.79E-01	1.54E-04
33	3	-9.63E-01	-9.94E-03	-1.00E+00	2.15E-07	9.77E-01	-1.14E-02
33	4	-8.51E-01	-5.58E-02	-4.91E-01	9.91E-04	1.85E-01	1.55E-01
33	5	1.66E-01	1.51E-01	8.97E-01	-8.08E-03	-8.97E-03	1.06E-01
34	1	-2.81E-01	9.36E-03	2.91E-01	3.58E-04	-3.00E-01	-1.28E-02

# CONTINUATION OF TABLE XXXII

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 1, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
34	2	6.93E-03	-1.05E-02	-3.02E-07	1.78E-08	-5.90E-03	6.20E-03
34	3	1.22E-02	9.11E-04	-1.45E-07	1.79E-08	6.64E-03	2.36E-03
34	4	5.91E-01	1.05E-01	4.02E-06	-1.47E-06	-5.64E-01	-1.37E-01
34	5	-5.43E-01	-1.30E-01	-1.00E+00	3.16E-08	-3.92E-01	-8.16E-02
35	1	-9.46E-01	6.02E-02	9.92E-01	-1.01E-03	-1.05E+00	-7.71E-02
35	2	-9.40E-01	-7.96E-03	9.89E-01	1.13E-03	9.52E-01	4.66E-02
35	3	-7.79E-01	-4.74E-02	9.75E-01	-1.83E-03	-9.16E-01	7.52E-03
35	4	1.52E-01	1.39E-01	-4.47E-01	3.51E-03	-7.43E-01	-7.89E-02
35	5	-2.88E-03	-1.04E-02	-8.19E-01	3.09E-03	-6.56E-01	-3.90E-02
36	1	-3.13E-01	1.29E-02	-9.92E-01	1.01E-03	9.77E-01	9.50E-03
36	2	-1.32E-02	1.84E-03	9.89E-01	1.13E-03	9.89E-01	9.48E-04
36	3	3.98E-01	-6.39E-02	9.75E-01	-1.83E-03	-9.38E-01	-3.67E-03
36	4	4.19E-01	-3.52E-02	-4.47E-01	3.51E-03	1.82E-01	1.43E-01
36	5	-9.53E-01	6.17E-02	8.19E-01	-3.09E-03	-1.78E-02	9.72E-02

#### **4.7.2 Results for 44-BAR Truss, 90 Model, Method 2**

**1) Method 2, Two-Mode System Identification:** The results of this computer run are documented in tables XXXIII and XXXIV. Table XXXIII indicates that the improvement of the first and second eigenvalues are 100 percent. Also, the eigenvelocities in table XXXIV show the same improvement, that is, nearly 100 percent improvement over the analytical results for the first and second modes. Overall, the results of the two-mode system identification appear quite satisfactory.

**2) Method 2, Five-Mode System Identification:** The results of this computer run are documented in tables XXXV and XXXVI. Table XXXV indicates that the improvements of the first, second, third, fourth, and fifth eigenvalues are near 100 percent. Also, the eigenvelocities in table XXXVI show the same improvements, that is, nearly 100 percent improvement over the analytical results for the first, second, third, forth, and fifth modes. Overall, the results of the five-mode system identification appear quite satisfactory.

**TABLE XXXIII**

**EIGENVALUES OF 44-BAR TRUSS  
2-MODE CORRELATION  
METHOD 2, 90 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.87E+01	1.19E+02
2	-1.91E+01	-1.40E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.95E+01	-1.20E+02
2	-1.98E+01	1.41E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.87E+01	-1.19E+02
2	-1.91E+01	1.40E+02
3	XXX	XXX
4	XXX	XXX
5	XXX	XXX

**TABLE XXXIV**

**EIGENVELOCITIES OF 44-BAR TRUSS  
2-MODE CORRELATION  
METHOD 2, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	2.57E-03	7.07E-03	-2.44E-08	8.15E-09	3.79E-03	-7.23E-03
1	2	2.24E-01	-7.86E-03	2.31E-01	8.63E-04	2.31E-01	8.82E-03
2	1	-4.63E-01	2.15E-02	4.66E-01	-2.54E-03	-4.61E-01	-2.25E-02
2	2	2.42E-01	-2.80E-02	2.23E-01	5.60E-04	2.28E-01	2.80E-02
3	1	4.74E-01	-6.50E-03	-4.66E-01	2.54E-03	4.68E-01	8.03E-03
3	2	2.16E-01	1.23E-02	2.23E-01	5.60E-04	2.16E-01	-1.15E-02
4	1	2.78E-01	-7.69E-03	-2.87E-01	1.50E-03	2.86E-01	8.82E-03
4	2	-4.06E-03	1.08E-02	5.21E-07	-1.60E-08	-3.99E-03	-1.12E-02
5	1	-4.59E-01	2.08E-02	4.62E-01	-3.10E-03	-4.58E-01	-2.17E-02
5	2	2.39E-01	-2.81E-02	2.20E-01	8.69E-04	2.26E-01	2.81E-02
6	1	4.61E-01	-3.17E-03	-4.62E-01	3.10E-03	4.65E-01	5.42E-03
6	2	2.28E-01	7.86E-03	2.20E-01	8.69E-04	2.13E-01	-8.49E-03
7	1	-2.03E-03	-7.16E-03	4.86E-08	-7.66E-09	-3.79E-03	7.27E-03
7	2	-2.24E-01	7.92E-03	-2.31E-01	-8.63E-04	-2.31E-01	-8.87E-03
8	1	-4.53E-01	1.87E-02	4.66E-01	-2.54E-03	-4.61E-01	-2.05E-02
8	2	2.26E-01	-2.40E-02	2.23E-01	5.60E-04	2.29E-01	2.54E-02
9	1	4.64E-01	-3.65E-03	-4.66E-01	2.54E-03	4.68E-01	5.92E-03
9	2	2.30E-01	8.19E-03	2.23E-01	5.60E-04	2.16E-01	-8.81E-03
10	1	-2.81E-01	7.90E-03	2.87E-01	-1.50E-03	-2.86E-01	-8.86E-03
10	2	7.48E-03	-1.10E-02	-5.61E-07	1.61E-08	3.98E-03	1.12E-02
11	1	-4.49E-01	1.80E-02	4.62E-01	-3.10E-03	-4.57E-01	-1.97E-02
11	2	2.24E-01	-2.41E-02	2.20E-01	8.69E-04	2.26E-01	2.54E-02
12	1	4.70E-01	-5.99E-03	-4.62E-01	3.10E-03	4.65E-01	7.52E-03
12	2	2.14E-01	1.19E-02	2.20E-01	8.69E-04	2.14E-01	-1.12E-02



# CONTINUATION OF TABLE XXXIV

## EIGENVELOCITIES OF 44-BAR TRUSS 2-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
13	1	5.14E-03	1.41E-02	-5.82E-08	1.70E-08	7.56E-03	-1.44E-02
13	2	4.45E-01	-1.51E-02	4.60E-01	1.08E-03	4.59E-01	1.70E-02
14	1	-8.78E-01	5.55E-02	8.89E-01	-3.96E-03	-8.73E-01	-5.76E-02
14	2	8.99E-01	-6.71E-02	8.78E-01	1.49E-03	8.88E-01	6.91E-02
15	1	9.07E-01	3.06E-04	-8.89E-01	3.96E-03	9.02E-01	2.20E-03
15	2	8.50E-01	8.79E-03	8.78E-01	1.49E-03	8.63E-01	-5.54E-03
16	1	3.35E-01	-1.05E-02	-3.45E-01	4.68E-04	3.44E-01	1.19E-02
16	2	-5.44E-03	1.28E-02	6.74E-07	-1.87E-08	-4.89E-03	-1.33E-02
17	1	-8.86E-01	5.66E-02	8.98E-01	-3.61E-03	-8.82E-01	-5.88E-02
17	2	9.10E-01	-6.75E-02	8.90E-01	1.11E-03	9.00E-01	6.96E-02
18	1	8.99E-01	5.38E-03	-8.98E-01	3.61E-03	9.10E-01	-1.40E-03
18	2	8.88E-01	1.58E-03	8.90E-01	1.11E-03	8.75E-01	-6.94E-04
19	1	-4.07E-03	-1.43E-02	-2.45E-08	-1.55E-08	-7.55E-03	1.45E-02
19	2	-4.47E-01	1.52E-02	-4.60E-01	-1.08E-03	-4.59E-01	-1.71E-02
20	1	-8.60E-01	5.05E-02	8.89E-01	-3.96E-03	-8.72E-01	-5.41E-02
20	2	8.71E-01	-5.98E-02	8.78E-01	1.49E-03	8.88E-01	6.44E-02
21	1	8.90E-01	5.59E-03	-8.89E-01	3.96E-03	9.01E-01	-1.65E-03
21	2	8.76E-01	1.33E-03	8.78E-01	1.49E-03	8.63E-01	-4.42E-04
22	1	-3.37E-01	1.08E-02	3.45E-01	-4.68E-04	-3.44E-01	-1.19E-02
22	2	8.79E-03	-1.31E-02	-6.67E-07	1.96E-08	4.88E-03	1.34E-02
23	1	-8.68E-01	5.14E-02	8.98E-01	-3.61E-03	-8.81E-01	-5.50E-02
23	2	8.83E-01	-6.00E-02	8.90E-01	1.11E-03	9.00E-01	6.46E-02
24	1	9.16E-01	-1.14E-04	-8.98E-01	3.61E-03	9.11E-01	2.63E-03
24	2	8.61E-01	9.32E-03	8.90E-01	1.11E-03	8.75E-01	-6.02E-03

# CONTINUATION OF TABLE XXXIV

## EIGENVELOCITIES OF 44-BAR TRUSS 2-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
25	1	5.14E-03	1.42E-02	1.75E-07	1.47E-08	7.61E-03	-1.45E-02
25	2	4.50E-01	-1.65E-02	4.65E-01	2.51E-03	4.64E-01	1.85E-02
26	1	-1.35E+00	9.77E-02	1.37E+00	3.91E-03	-1.34E+00	-1.01E-01
26	2	1.48E+00	-9.73E-02	1.46E+00	-4.70E-03	1.48E+00	1.02E-01
27	1	1.39E+00	-5.61E-03	-1.37E+00	-3.91E-03	1.39E+00	8.98E-03
27	2	1.41E+00	1.53E-02	1.46E+00	-4.70E-03	1.44E+00	-9.84E-03
28	1	3.88E-01	-1.43E-02	-4.00E-01	-1.63E-03	3.99E-01	1.58E-02
28	2	-6.79E-03	1.47E-02	2.74E-07	-2.69E-08	-5.79E-03	-1.52E-02
29	1	-1.34E+00	9.53E-02	1.36E+00	2.50E-03	-1.33E+00	-9.88E-02
29	2	1.46E+00	-9.79E-02	1.44E+00	-3.00E-03	1.46E+00	1.03E-01
30	1	1.36E+00	2.40E-03	-1.36E+00	-2.50E-03	1.38E+00	2.98E-03
30	2	1.42E+00	4.83E-03	1.44E+00	-3.00E-03	1.42E+00	-2.05E-03
31	1	-4.07E-03	-1.44E-02	-5.10E-07	-1.52E-08	-7.60E-03	1.46E-02
31	2	-4.51E-01	1.67E-02	-4.65E-01	-2.51E-03	-4.64E-01	-1.86E-02
32	1	-1.33E+00	9.11E-02	1.37E+00	3.91E-03	-1.34E+00	-9.67E-02
32	2	1.45E+00	-8.87E-02	1.46E+00	-4.70E-03	1.48E+00	9.61E-02
33	1	1.37E+00	1.29E-03	-1.37E+00	-3.91E-03	1.39E+00	4.16E-03
33	2	1.44E+00	6.22E-03	1.46E+00	-4.70E-03	1.44E+00	-3.39E-03
34	1	-3.91E-01	1.46E-02	4.00E-01	1.63E-03	-3.99E-01	-1.59E-02
34	2	1.00E-02	-1.50E-02	-4.41E-07	2.73E-08	5.78E-03	1.54E-02
35	1	-1.31E+00	8.90E-02	1.36E+00	2.50E-03	-1.33E+00	-9.46E-02
35	2	1.43E+00	-8.97E-02	1.44E+00	-3.00E-03	1.46E+00	9.69E-02
36	1	1.38E+00	-4.64E-03	-1.36E+00	-2.50E-03	1.38E+00	7.99E-03
36	2	1.39E+00	1.41E-02	1.44E+00	-3.00E-03	1.42E+00	-8.71E-03

**TABLE XXXV**

**EIGENVALUES OF 44-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 90 MODEL**

TEST EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.87E+01	1.19E+02
2	-1.91E+01	-1.40E+02
3	-1.85E+01	-2.07E+02
4	-1.47E+01	-3.96E+02
5	-1.56E+01	3.99E+02

ANALYSIS EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.95E+01	-1.20E+02
2	-1.98E+01	1.41E+02
3	-1.93E+01	-2.12E+02
4	-1.51E+01	-4.06E+02
5	-1.62E+01	4.02E+02

CORRELATED EIGENVALUES IN Hz		
MODE	REAL	IMAGINARY
1	-1.89E+01	-1.19E+02
2	-1.93E+01	1.40E+02
3	-1.87E+01	2.07E+02
4	-1.47E+01	3.96E+02
5	-1.55E+01	-3.99E+02

TABLE XXXVI

**EIGENVELOCITIES OF 44-BAR TRUSS  
5-MODE CORRELATION  
METHOD 2, 90 MODEL**

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
1	1	3.72E-03	1.14E-02	-4.18E-08	1.42E-08	4.17E-03	-8.92E-03
1	2	3.56E-01	-8.33E-04	3.96E-01	-9.41E-04	3.63E-01	1.81E-04
1	3	1.07E-02	-1.99E-03	2.89E-09	4.14E-08	9.52E-03	1.15E-03
1	4	-4.39E-01	8.23E-02	-6.93E-01	1.78E-02	-4.46E-01	-8.46E-02
1	5	-4.60E-01	6.43E-02	-3.21E-07	-2.22E-07	-4.79E-01	-6.68E-02
2	1	-7.38E-01	1.01E-02	7.99E-01	-9.23E-03	-7.44E-01	-5.31E-03
2	2	3.86E-01	-3.20E-02	3.82E-01	-1.37E-03	3.92E-01	2.37E-02
2	3	-9.25E-01	4.47E-03	-1.00E+00	1.20E-02	-9.37E-01	-1.50E-03
2	4	5.05E-01	-3.99E-01	2.21E+00	-1.38E-03	5.00E-01	3.93E-01
2	5	2.39E+00	-1.07E-01	1.34E+00	-3.19E-02	2.37E+00	1.03E-01
3	1	7.54E-01	1.44E-02	-7.99E-01	9.23E-03	7.63E-01	-1.50E-02
3	2	3.43E-01	3.08E-02	3.82E-01	-1.37E-03	3.47E-01	-2.36E-02
3	3	9.10E-01	-7.63E-03	1.00E+00	-1.20E-02	9.17E-01	2.45E-03
3	4	2.33E+00	-7.13E-02	2.21E+00	-1.38E-03	2.33E+00	7.95E-02
3	5	5.93E-01	-3.79E-01	-1.34E+00	3.19E-02	6.57E-01	3.93E-01
4	1	4.43E-01	2.29E-03	-4.92E-01	5.57E-03	4.57E-01	-3.15E-03
4	2	-7.02E-03	1.69E-02	8.92E-07	-3.29E-08	-7.23E-03	-1.35E-02
4	3	-4.83E-03	-1.32E-03	-8.27E-07	2.00E-10	-4.24E-03	3.82E-04
4	4	-4.35E-02	1.04E-03	-1.96E-07	1.19E-07	-3.17E-02	-4.06E-03
4	5	4.36E-02	1.59E-02	4.19E-02	1.25E-02	2.60E-02	-1.32E-02
5	1	-7.32E-01	9.02E-03	7.93E-01	-1.02E-02	-7.39E-01	-4.26E-03
5	2	3.82E-01	-3.23E-02	3.78E-01	-8.17E-04	3.87E-01	2.40E-02
5	3	-9.05E-01	6.53E-03	-9.77E-01	1.40E-02	-9.15E-01	-3.66E-03
5	4	4.64E-01	-3.72E-01	2.03E+00	-1.44E-02	4.56E-01	3.65E-01

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS

### 5-MODE CORRELATION

### METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
5	5	2.20E+00	-8.52E-02	1.23E+00	-2.21E-02	2.18E+00	8.12E-02
6	1	7.34E-01	1.90E-02	-7.93E-01	1.02E-02	7.41E-01	-1.73E-02
6	2	3.62E-01	2.44E-02	3.78E-01	-8.17E-04	3.67E-01	-2.02E-02
6	3	-9.12E-01	2.17E-03	-9.77E-01	1.40E-02	-9.18E-01	-3.60E-03
6	4	2.15E+00	-8.93E-02	2.03E+00	-1.44E-02	2.14E+00	8.35E-02
6	5	5.96E-01	-3.50E-01	-1.23E+00	2.21E-02	5.92E-01	3.57E-01
7	1	-2.86E-03	-1.15E-02	8.32E-08	-1.36E-08	-4.17E-03	8.95E-03
7	2	-3.57E-01	8.91E-04	-3.96E-01	9.41E-04	-3.63E-01	-2.29E-04
7	3	-1.30E-02	1.71E-03	-1.47E-08	-3.77E-08	-9.58E-03	-1.01E-03
7	4	4.40E-01	-8.53E-02	6.93E-01	-1.78E-02	4.46E-01	8.51E-02
7	5	4.74E-01	-6.33E-02	5.79E-07	2.30E-07	4.79E-01	6.67E-02
8	1	-7.22E-01	6.15E-03	7.99E-01	-9.23E-03	-7.28E-01	-3.71E-03
8	2	3.62E-01	-2.63E-02	3.82E-01	-1.37E-03	3.67E-01	2.10E-02
8	3	9.11E-01	-1.19E-03	1.00E+00	-1.20E-02	9.19E-01	4.74E-04
8	4	5.00E-01	-3.85E-01	2.21E+00	-1.38E-03	5.03E-01	3.96E-01
8	5	2.32E+00	-1.01E-01	1.34E+00	-3.19E-02	2.38E+00	1.04E-01
9	1	7.39E-01	1.85E-02	-7.99E-01	9.23E-03	7.47E-01	-1.68E-02
9	2	3.66E-01	2.51E-02	3.82E-01	-1.37E-03	3.72E-01	-2.08E-02
9	3	-9.31E-01	5.75E-04	-1.00E+00	1.20E-02	-9.40E-01	-2.04E-03
9	4	2.33E+00	-8.46E-02	2.21E+00	-1.38E-03	2.33E+00	7.90E-02
9	5	6.45E-01	-3.83E-01	-1.34E+00	3.19E-02	6.44E-01	3.92E-01
10	1	-4.47E-01	-2.10E-03	4.92E-01	-5.57E-03	-4.57E-01	3.10E-03
10	2	1.25E-02	-1.71E-02	-9.61E-07	3.35E-08	7.27E-03	1.36E-02
10	3	1.83E-02	1.55E-03	7.29E-07	5.15E-10	4.24E-03	-7.50E-04

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
10	4	5.02E-02	-6.55E-03	1.17E-07	-1.18E-07	3.19E-02	4.43E-03
10	5	-1.27E-02	-1.54E-02	-4.19E-02	-1.25E-02	-2.56E-02	1.31E-02
11	1	-7.16E-01	5.16E-03	7.93E-01	-1.02E-02	-7.23E-01	-2.69E-03
11	2	3.58E-01	-2.66E-02	3.78E-01	-8.17E-04	3.63E-01	2.13E-02
11	3	8.92E-01	-3.29E-03	9.77E-01	-1.40E-02	9.00E-01	2.59E-03
11	4	4.60E-01	-3.58E-01	2.03E+00	-1.44E-02	4.59E-01	3.68E-01
11	5	2.14E+00	-8.02E-02	1.23E+00	-2.21E-02	2.19E+00	8.14E-02
12	1	7.49E-01	1.50E-02	-7.93E-01	1.02E-02	7.57E-01	-1.56E-02
12	2	3.40E-01	3.02E-02	3.78E-01	-8.17E-04	3.42E-01	-2.30E-02
12	3	8.91E-01	-9.03E-03	9.77E-01	-1.40E-02	8.97E-01	4.00E-03
12	4	2.15E+00	-7.70E-02	2.03E+00	-1.44E-02	2.14E+00	8.42E-02
12	5	5.48E-01	-3.47E-01	-1.23E+00	2.21E-02	6.05E-01	3.58E-01
13	1	7.45E-03	2.27E-02	-9.96E-08	2.97E-08	8.41E-03	-1.78E-02
13	2	7.09E-01	-7.29E-04	7.89E-01	-2.96E-03	7.24E-01	-5.92E-04
13	3	2.11E-02	-3.89E-03	8.65E-09	8.36E-08	1.86E-02	2.22E-03
13	4	-8.45E-01	1.56E-01	-1.33E+00	3.00E-02	-8.56E-01	-1.60E-01
13	5	-8.84E-01	1.26E-01	-6.49E-07	-4.22E-07	-9.19E-01	-1.31E-01
14	1	-1.40E+00	4.25E-02	1.52E+00	-1.61E-02	-1.41E+00	-2.81E-02
14	2	1.43E+00	-5.98E-02	1.51E+00	-6.62E-03	1.45E+00	4.30E-02
14	3	-1.57E+00	1.01E-03	-1.72E+00	1.77E-02	-1.58E+00	1.86E-03
14	4	-3.31E-01	-2.60E-01	9.44E-01	1.22E-04	-3.60E-01	2.58E-01
14	5	1.54E+00	4.18E-02	1.40E+00	-2.31E-02	1.53E+00	-5.26E-02
15	1	1.44E+00	4.79E-02	-1.52E+00	1.61E-02	1.46E+00	-4.34E-02
15	2	1.35E+00	5.84E-02	1.51E+00	-6.62E-03	1.36E+00	-4.61E-02

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
15	3	1.59E+00	-1.45E-02	1.72E+00	-1.77E-02	1.60E+00	4.51E-03
15	4	1.54E+00	5.94E-02	9.44E-01	1.18E-04	1.56E+00	-5.82E-02
15	5	-2.66E-01	-2.53E-01	-1.40E+00	2.31E-02	-2.47E-01	2.66E-01
16	1	5.33E-01	7.15E-04	-5.92E-01	4.41E-03	5.46E-01	-1.44E-03
16	2	-9.32E-03	2.01E-02	1.15E-06	-3.91E-08	-9.90E-03	-1.60E-02
16	3	-8.47E-03	-1.65E-03	-8.69E-07	-1.08E-08	-9.36E-03	1.01E-03
16	4	-7.50E-01	-1.02E-01	-4.77E-06	1.80E-06	-7.78E-01	1.01E-01
16	5	7.27E-01	1.46E-01	1.12E+00	2.29E-02	7.21E-01	-1.55E-01
17	1	-1.41E+00	4.39E-02	1.54E+00	-1.56E-02	-1.43E+00	-2.93E-02
17	2	1.45E+00	-5.99E-02	1.53E+00	-7.40E-03	1.47E+00	4.29E-02
17	3	-1.61E+00	1.05E-04	-1.78E+00	1.68E-02	-1.63E+00	2.93E-03
17	4	-3.68E-01	-2.92E-01	1.06E+00	4.69E-03	-4.00E-01	2.91E-01
17	5	1.71E+00	3.98E-02	1.57E+00	-3.15E-02	1.72E+00	-5.19E-02
18	1	1.43E+00	5.55E-02	-1.54E+00	1.56E-02	1.44E+00	-4.67E-02
18	2	1.41E+00	4.89E-02	1.53E+00	-7.40E-03	1.43E+00	-4.19E-02
18	3	-1.63E+00	-8.20E-03	-1.78E+00	1.68E-02	-1.64E+00	3.90E-03
18	4	1.71E+00	7.36E-02	1.06E+00	4.68E-03	1.74E+00	-7.81E-02
18	5	-3.13E-01	-2.86E-01	-1.57E+00	3.15E-02	-3.03E-01	2.99E-01
19	1	-5.73E-03	-2.29E-02	-4.22E-08	-2.62E-08	-8.40E-03	1.78E-02
19	2	-7.12E-01	8.41E-04	-7.89E-01	2.96E-03	-7.24E-01	4.95E-04
19	3	-2.58E-02	3.33E-03	-2.33E-07	-7.14E-08	-1.87E-02	-1.95E-03
19	4	8.47E-01	-1.62E-01	1.33E+00	-3.00E-02	8.56E-01	1.61E-01
19	5	9.11E-01	-1.25E-01	8.76E-07	4.42E-07	9.20E-01	1.31E-01
20	1	-1.37E+00	3.54E-02	1.52E+00	-1.61E-02	-1.38E+00	-2.53E-02

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS

### 5-MODE CORRELATION

### METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
20	2	1.39E+00	-4.97E-02	1.51E+00	-6.62E-03	1.40E+00	3.82E-02
20	3	1.60E+00	-3.09E-03	1.72E+00	-1.77E-02	1.62E+00	4.12E-04
20	4	-3.19E-01	-2.59E-01	9.44E-01	1.22E-04	-3.54E-01	2.66E-01
20	5	1.53E+00	4.37E-02	1.40E+00	-2.31E-02	1.56E+00	-5.26E-02
21	1	1.42E+00	5.54E-02	-1.52E+00	1.61E-02	1.43E+00	-4.66E-02
21	2	1.39E+00	4.79E-02	1.51E+00	-6.62E-03	1.41E+00	-4.09E-02
21	3	-1.58E+00	-6.61E-03	-1.72E+00	1.77E-02	-1.59E+00	2.47E-03
21	4	1.53E+00	6.15E-02	9.44E-01	1.18E-04	1.55E+00	-6.52E-02
21	5	-2.83E-01	-2.56E-01	-1.40E+00	2.31E-02	-2.73E-01	2.66E-01
22	1	-5.37E-01	-4.47E-04	5.92E-01	-4.41E-03	-5.46E-01	1.36E-03
22	2	1.47E-02	-2.04E-02	-1.14E-06	4.05E-08	9.97E-03	1.61E-02
22	3	2.16E-02	1.73E-03	4.82E-07	1.41E-08	9.32E-03	-1.20E-03
22	4	7.59E-01	9.68E-02	4.89E-06	-1.81E-06	7.78E-01	-1.01E-01
22	5	-6.98E-01	-1.45E-01	-1.12E+00	-2.29E-02	-7.21E-01	1.54E-01
23	1	-1.39E+00	3.64E-02	1.54E+00	-1.56E-02	-1.40E+00	-2.62E-02
23	2	1.41E+00	-4.94E-02	1.53E+00	-7.40E-03	1.42E+00	3.78E-02
23	3	1.65E+00	-2.20E-03	1.78E+00	-1.68E-02	1.67E+00	-5.73E-04
23	4	-3.55E-01	-2.90E-01	1.06E+00	4.69E-03	-3.94E-01	2.99E-01
23	5	1.71E+00	4.24E-02	1.57E+00	-3.15E-02	1.75E+00	-5.20E-02
24	1	1.46E+00	4.77E-02	-1.54E+00	1.56E-02	1.47E+00	-4.32E-02
24	2	1.37E+00	5.98E-02	1.53E+00	-7.40E-03	1.38E+00	-4.74E-02
24	3	1.64E+00	-1.25E-02	1.78E+00	-1.68E-02	1.65E+00	2.28E-03
24	4	1.72E+00	7.42E-02	1.06E+00	4.68E-03	1.75E+00	-7.21E-02



# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS

### 5-MODE CORRELATION

### METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
24	5	-3.04E-01	-2.82E-01	-1.57E+00	3.15E-02	-2.77E-01	2.98E-01
25	1	7.45E-03	2.29E-02	2.99E-07	2.34E-08	8.25E-03	-1.79E-02
25	2	7.16E-01	-2.82E-03	7.98E-01	-5.55E-04	7.31E-01	1.53E-03
25	3	2.16E-02	-4.08E-03	3.07E-07	7.79E-08	1.96E-02	2.38E-03
25	4	-9.22E-01	1.76E-01	-1.46E+00	4.35E-02	-9.38E-01	-1.82E-01
25	5	-9.67E-01	1.31E-01	-3.89E-07	-4.63E-07	-1.01E+00	-1.37E-01
26	1	-2.15E+00	8.52E-02	2.35E+00	-7.65E-03	-2.17E+00	-6.25E-02
26	2	2.36E+00	-7.76E-02	2.50E+00	-2.33E-02	2.38E+00	5.27E-02
26	3	-1.88E+00	-1.85E-02	-2.11E+00	1.14E-03	-1.89E+00	1.93E-02
26	4	5.52E-01	3.22E-01	-1.08E+00	2.86E-03	5.83E-01	-3.17E-01
26	5	-2.01E+00	-1.01E-01	-1.87E+00	-2.08E-03	-1.98E+00	1.12E-01
27	1	2.21E+00	6.37E-02	-2.35E+00	7.65E-03	2.23E+00	-5.42E-02
27	2	2.24E+00	9.78E-02	2.50E+00	-2.33E-02	2.25E+00	-8.01E-02
27	3	1.95E+00	-3.56E-03	2.11E+00	-1.14E-03	1.97E+00	-8.36E-03
27	4	-1.99E+00	-8.51E-02	-1.08E+00	2.86E-03	-2.00E+00	7.58E-02
27	5	5.28E-01	3.42E-01	1.87E+00	2.08E-03	4.70E-01	-3.57E-01
28	1	6.19E-01	-2.44E-03	-6.86E-01	1.39E-03	6.31E-01	1.88E-03
28	2	-1.16E-02	2.31E-02	4.70E-07	-4.91E-08	-1.24E-02	-1.83E-02
28	3	-1.19E-02	-1.98E-03	2.76E-08	-2.25E-08	-1.39E-02	1.64E-03
28	4	-1.38E+00	-2.00E-01	-8.84E-06	3.25E-06	-1.44E+00	2.02E-01
28	5	1.33E+00	2.56E-01	2.09E+00	2.11E-02	1.34E+00	-2.73E-01
29	1	-2.13E+00	8.20E-02	2.33E+00	-9.96E-03	-2.15E+00	-5.96E-02
29	2	2.33E+00	-7.94E-02	2.47E+00	-2.02E-02	2.35E+00	5.49E-02
29	3	-1.84E+00	-1.53E-02	-2.06E+00	4.96E-03	-1.85E+00	1.61E-02

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
29	4	5.07E-01	2.91E-01	-9.86E-01	8.36E-03	5.34E-01	-2.86E-01
29	5	-1.84E+00	-1.02E-01	-1.71E+00	-1.08E-02	-1.80E+00	1.12E-01
30	1	2.16E+00	7.48E-02	-2.33E+00	9.96E-03	2.18E+00	-5.95E-02
30	2	2.27E+00	8.21E-02	2.47E+00	-2.02E-02	2.28E+00	-7.11E-02
30	3	-1.88E+00	-2.74E-02	-2.06E+00	4.96E-03	-1.90E+00	2.14E-02
30	4	-1.82E+00	-4.47E-02	-9.86E-01	8.37E-03	-1.84E+00	4.64E-02
30	5	3.78E-01	3.16E-01	1.71E+00	1.08E-02	3.94E-01	-3.28E-01
31	1	-5.72E-03	-2.31E-02	-8.74E-07	-2.07E-08	-8.24E-03	1.80E-02
31	2	-7.19E-01	2.94E-03	-7.98E-01	5.55E-04	-7.31E-01	-1.63E-03
31	3	-2.64E-02	3.52E-03	7.48E-08	-6.69E-08	-1.97E-02	-2.11E-03
31	4	9.24E-01	-1.83E-01	1.46E+00	-4.35E-02	9.38E-01	1.83E-01
31	5	9.97E-01	-1.29E-01	1.33E-06	4.96E-07	1.01E+00	1.36E-01
32	1	-2.11E+00	7.58E-02	2.35E+00	-7.65E-03	-2.13E+00	-5.90E-02
32	2	2.31E+00	-6.57E-02	2.50E+00	-2.33E-02	2.33E+00	4.71E-02
32	3	2.00E+00	1.22E-02	2.11E+00	-1.14E-03	2.02E+00	-1.72E-02
32	4	5.57E-01	2.95E-01	-1.08E+00	2.86E-03	5.91E-01	-3.03E-01
32	5	-1.89E+00	-1.02E-01	-1.87E+00	-2.08E-03	-1.94E+00	1.13E-01
33	1	2.18E+00	7.36E-02	-2.35E+00	7.65E-03	2.20E+00	-5.79E-02
33	2	2.29E+00	8.51E-02	2.50E+00	-2.33E-02	2.31E+00	-7.42E-02
33	3	-1.93E+00	-3.16E-02	-2.11E+00	1.14E-03	-1.95E+00	2.58E-02
33	4	-1.99E+00	-5.82E-02	-1.08E+00	2.86E-03	-2.01E+00	6.12E-02
33	5	4.12E-01	3.44E-01	1.87E+00	2.08E-03	4.34E-01	-3.58E-01
34	1	-6.23E-01	2.80E-03	6.86E-01	-1.39E-03	-6.31E-01	-1.99E-03

# CONTINUATION OF TABLE XXXVI

## EIGENVELOCITIES OF 44-BAR TRUSS 5-MODE CORRELATION METHOD 2, 90 MODEL

D.O.F	MODE	TEST		ANALYSIS		CORRELATED	
		REAL	IMAGINARY	REAL	IMAGINARY	REAL	IMAGINARY
34	2	1.68E-02	-2.34E-02	-7.56E-07	5.15E-08	1.25E-02	1.84E-02
34	3	2.43E-02	1.97E-03	-3.05E-07	3.79E-08	1.39E-02	-1.64E-03
34	4	1.39E+00	1.96E-01	8.88E-06	-3.26E-06	1.44E+00	-2.02E-01
34	5	-1.31E+00	-2.54E-01	-2.09E+00	-2.11E-02	-1.34E+00	2.73E-01
35	1	-2.10E+00	7.30E-02	2.33E+00	-9.96E-03	-2.11E+00	-5.63E-02
35	2	2.28E+00	-6.80E-02	2.47E+00	-2.02E-02	2.30E+00	4.95E-02
35	3	1.95E+00	8.92E-03	2.06E+00	-4.96E-03	1.97E+00	-1.37E-02
35	4	5.12E-01	2.67E-01	-9.86E-01	8.36E-03	5.42E-01	-2.74E-01
35	5	-1.72E+00	-1.03E-01	-1.71E+00	-1.08E-02	-1.77E+00	1.13E-01
36	1	2.20E+00	6.47E-02	-2.33E+00	9.96E-03	2.21E+00	-5.54E-02
36	2	2.21E+00	9.51E-02	2.47E+00	-2.02E-02	2.23E+00	-7.75E-02
36	3	1.91E+00	-7.01E-03	2.06E+00	-4.96E-03	1.92E+00	-4.85E-03
36	4	-1.82E+00	-6.97E-02	-9.86E-01	8.37E-03	-1.83E+00	6.13E-02
36	5	4.79E-01	3.14E-01	1.71E+00	1.08E-02	4.28E-01	-3.27E-01

## 4.8 Concluding Remarks

The result of the system identification using equation (3.34) appeared satisfactory when a small number of modes were selected. However, as the number of modes increased, the quality of the eigenvalues and eigenvelocities deteriorated. The reason for this degradation is that the differences between the modal deformations of the higher modes of the test model and the analytical model are higher than those of the lower modes. In fact, generally speaking, higher modes, even if they are system level, would contain some level of local modes. Because of the severe local differences between the test model and the analytical modes, the local modes of the two models are extremely different. As a result, these extreme local differences in modal deformations will corrupt the final result. Overall, the system identification using method 1 provides successful results with one to three modes. It is recommended, however, to use more modes, because the first mode does not represent the entire structural system from the kinetic energy standpoint.

As a result of system identification using method 1, the test damping matrix becomes complex. It was discovered, by observation, when applying method 1 to the 12-BAR truss and the 44-BAR truss, that even for highly damped structural systems, the imaginary portion of the test damping matrix is much smaller (two to three orders of magnitude) than the real portion. As a test, the system identification was repeated without the imaginary portion of the test damping matrix. The results of these computer runs were very similar and sometimes identical to those done with the entire test damping matrix. Therefore, the option in method 1 that eliminates the imaginary portion of the test damping matrix was left as part of the FORTRAN program. Because the results of the system identification using a full complex test damping matrix are so similar to those using the real portion of the test damping matrix, no tables or figures are provided here.

The result of the system identification using equations (3.47), and (3.48) appeared satisfactory when any number (small or large) of modes was selected. The computer runs for the class of structural systems in this study required two to fifteen iterations to converge. A total of sixteen computer runs for both the 12-BAR truss and the 44-BAR truss were made. Two of these computer runs did not converge. These nonconverging computer runs were the 95 model of the 12-BAR truss using two modes, and the 95 model of the 44-BAR truss using three modes. There were some other computer runs that did not converge at the beginning, but when the convergence criteria were loosened up, they converged. Overall, the system identification using method 2 provides successful results with one to five modes. It is recommended, however, to use more than one mode, since the first mode does not represent the entire structural system from the kinetic energy standpoint. With an increase in the number of modes, however, the number of iterations and the computer usage time increases. It is recommended to

use only the important modes with the highest kinetic energy when using method 2 to cut down the number of iterations and eliminate the possibility of nonconvergence. Also in the case of nonconvergence, it is recommended to use a smaller number of modes rather than loosening up the convergence criteria because when the convergence criteria is loosened up, the quality of the eigenvalues and eigenvectors degrade for all modes.

When using a small number of modes, both methods 1 and 2 were equally successful in correlating the eigenvalues, and the correlated eigenvalues are within a small percentage, most often less than one percent, of the test eigenvalues.

When a large number of modes (more than three modes) were selected for the system identification, method 2 provided accurate results, and method 1 converged to incorrect results. The reason for the higher success of method 2 is that this method is based on an iterative technique given a set of convergence criteria. This means that if the desired solution (eigenvalues and eigenvelocities) are not reached, the FORTRAN program changes the damping coefficients by some tolerance and repeats the problem until an acceptable solution is achieved. The disadvantage of method 2 over method 1 is that it requires more computer usage time. Each iteration of method 2 requires approximately as much computer usage time as that required during all phases of method 1.

Method 1 appeared to be more successful than method 2 when a small number of modes was selected for the system identification. The success here is defined as the quality and closeness of the eigenvelocities, in other words, the correlated eigenvelocities came closer to the test eigenvelocities using method 1. This success and better performance of method 1 can be attributed to the closed form solution of the equation for the damping matrix. Perhaps if the convergence criteria for method 2 considered the differences between the eigenvelocity vectors (as opposed to kinetic energy values) of the test model and the correlated model, method 2 would be more successful than method 1. This convergence criteria would definitely be more stringent, because the size of the eigenvelocity matrix is  $n$ -by- $m$  as opposed to  $m$ -by- $m$  of the kinetic energy matrix. Also, the kinetic energy matrix is a diagonal matrix, which means that there are less terms ( $m$  terms) in the matrix to deal with than the number of terms consisting the eigenvelocity matrix ( $n*m$  terms). The disadvantages of making the convergence criteria in method 2 more stringent in order to achieve better quality eigenvelocities are the increase in number of iterations, increase in computer usage time, and increase in the possibility of non-convergence.

Overall, method 1 is somewhat better than method 2 because of the following:

- 1) It does not need convergence criteria, thus, the analyst does not have to spend time on setting convergence criteria.

- 2) It requires only one iteration, thus it takes less computer usage time to execute.
- 3) The test eigenvelocities of method 1 are closer to the test eigenvelocities. Therefore, the eigenvectors of method 1 are more accurate to use in decoupling the equation of motion for load analysis when either the mode displacement method or the mode acceleration method are used.

## **CHAPTER V**

### **CONCLUSIONS AND FUTURE WORK**

Two approaches toward structural modification were proposed to correct the nonproportional damping matrix of a truss structure. Several formulations for both methods were derived. Method 1 uses the weighted difference between the test damping matrix and the analytical damping matrix as the basis for a minimization problem with the orthogonality condition as the constraint equation. Method 2 uses the weighted difference between the test damping matrix and the analytical damping matrix, as well as the weighted difference between the test stiffness matrix and the analytical stiffness matrix, as the basis for a multi-objective minimization problem with the equation of motion as the constraint equation.

Although the formulations of method 1 of this study modify only the damping matrix, this method can also be applied to structures whose stiffness matrix contains inherent inaccuracies. In fact, the same structures (with the same spatial properties) were used for both methods to examine method 1 for structures with inaccuracies in both the damping matrix and the stiffness matrix, and to compare the results of method 1 with those of method 2.

A FORTRAN program was developed which uses the formulations of methods 1 and 2 for system identification of a structural system, which can either read the user-generated spatial matrix data or interface with MSC/NASTRAN OUTPUT4 format data. The FORTRAN program can also provide simulated data to replace actual test data and extract the complex eigenvalues and eigenvectors of structural systems possessing rigid body motion.

The entire methodology, including formulations for both methods, was developed and checked using a simple four-degree-of-freedom system. Then the methods were applied to two classes of structures, namely, a plane truss and a space truss. The spatial property matrices of both trusses were varied in such a manner that the axial, bending, and (in the case of the space truss) torsional modes would couple. Up to 50 percent of the coefficients of the spatial property matrices were varied by up to 10 percent of the value of the analytical matrices. The results of this study showed nearly 100 percent improvement of the correlated eigensystem over the analytical eigensystem.

The performances of the methods developed in this study were compared, and it was concluded that method 1 is a better method to use for system identification when up to three modes are selected for the process. Also in this comparison, it was concluded that method 2 can handle a larger number of modes in system identification. However,

the computer usage time and the analyst's time needed to set up and test convergence criteria with each problem when using method 2 would require extra effort.

The results of this study for a simple four-degree-of-freedom system, a plane truss structure, and a space truss structure prove that the objectives of this dissertation, as stated in the Introduction, have been fully achieved.

Only simulated data were used in this study and although the author made a large number of changes in the spatial property matrices of the analytical model in order to simulate the test data, the real test of the methods proposed in this study will be their implementation in an actual modal survey test. The methodology, formulations, and FORTRAN program developed in this study should be applied to the measured data from an actual modal survey test of a structure that possesses complex modes. Future work relating to this study should consist of both the implementation of the proposed formulations to a modal survey test and the development of more accurate analytical methods to solve the set of governing equations ((3.47) and (3.48)) in method 2.



## **BIBLIOGRAPHY**

- Baruch, M., Itshack, Y.B., **"Optimal Weighted Orthogonalization of Measured Modes,"** AIAA Journal, Vol. 16, April 1978, pp. 346–351.
- Baruch, M., **"Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Tests,"** AIAA Journal, Vol.21, November 1978, pp. 1208–1210.
- Baruch, M., Itzhack, Y., **"Optimal Weighted Orthogonalization of Measured Modes,"**. TAE Report 297 (1977).
- Berman, A., **"Mass Matrix Correction Using an Incomplete set of Measured Modes,"** AIAA Journal, Vol. 17, October 1979, pp. 1147– 1148.
- Berman, A., Nagy, E.J., **"Improvement of a Large Analytical Model Using Test Data,"** AIAA Journal, Vol. 21, August 1983, pp. 1168– 1173.
- Berman, A., **"Improved Orthogonality Check for Measured Modes,"** AIAA Journal, Vol. 18, September 1980, pp. 1151–1152.
- Berman, A., Flannelly, W.G., **"Theory of Incomplete Models of Dynamic Structure,"** AIAA Journal, Vol. 9, August 1971, pp. 1482– 1487.
- Berman, A., **"System Identification Of A Complex Structure,"**. AIAA Paper 75–809, May 1975, pp. 1–6.
- Brock, J.E., **"Optimal Matrices Describing Linear Systems,"** AIAA Journal, Vol.6, July 1968, pp. 1292–1296.
- Businger, P.A., Gloub, G.H., **"Algorithm 358 – Singular Value Decomposition Of A Complex Matrix [F1,4,5],"**. Comm. Of The ACM, Vol. 12, No. 10, October 1969, pp. 564–565.
- Caesar, B., **"Update and Identification of Dynamic Mathematical Models,"**. First IMAC Conf., 1983, pp. 394–401.
- Caesar, B., **"Updating System Matrices Using Modal Test Data,"**. Fifth IMAC Conf., 1987, pp. 453–459.

Carneiro, S.H.S., Arruda, J.R.F., **"Updating Mechanical Joint Properties Based on Experimental Determined Modal Parameters,"** Fifth IMAC, 1987, pp. 1169–1174.

Chan, T.F., **"Algorithm 581, An Improved Algorithm for Computing the Singular Value Decomposition,"** ACM Transaction on Mathematical Software, Vol. 8, No. 1, March 1982, pp. 85–88.

Chen, J.C., Garba, J.A., **"Analytical Model Improvement Using Model Test Results,"** AIAA Journal, Vol. 18, June 1980, pp. 684–690.

Chen, J.C., Kuo, C.P, Garba, J.A., **"Direct Structural Parameter Identification by Modal Test Results,"** AIAA Paper 83–0812, May 1983, pp. 44–49.

Chen, J.C., Peretti, L.F, Garba, J.A., **"Spacecraft Structural System Identification by Modal Test,"** AIAA Paper 84–1051, May 1984, pp. 478–489.

Chen, J.C., Wada B.K., **"Criteria for Analysis–Test Correlation of Structural Dynamic Systems,"** Journal of Applied Mechanics, June 1975, pp. 471–477.

Chen, J.C., **"Evaluation of Modal Testing Methods,"** AIAA Paper 84–1071, 1984, pp. 561–572.

Chen, J.C., **"Analytical Model Accuracy Requirements for Structural Dynamic Systems,"** AIAA Journal, Vol. 21, July 1984, pp. 366–373.

Chen, J.C., Garba, J.A., **"Structural Analysis Model Validation Using Modal Test Data,"**

Chung, Y.T., Simonian, S.S., **"Test Analysis Model Correlation For The Gamma Ray Observatory,"** The International Journal of Analytical and Experimental Modal Analysis 6(3):161–173, July 1991.

Clough, R.W., Penzien, J., **"Dynamics of Structures,"** McGraw–Hill, 1975.

Collins, J.D., Hart, G.C., Hasselman, T.K., Kennedy, B., **"Statistical Identification of Structures,"** AIAA Journal, Vol. 12, February 1974, pp. 185–190.

Dodoson, B.J., **“Modification of Finite Element Model Using Experimental Modal Analysis,”** Second IMAC Conf., 1984, pp. 593– 601.

Ewins, D.J., **“Modal Testing of machinery Components,”** Imperial College of Science & Technology, London, UK.

Ewins, D.J., **“Applications and Advances of Modal Testing,”** Imperial College of Science & Technology, London, UK.

Ewins, D.J., **“Modal Testing Techniques,”** Imperial College of Science & Technology, London, UK.

Fillod R., Lauement, G., **“Global Method Of Modal Identification,”** Third IMAC, 1985, pp. 1145–1151.

Grossman, D.T., **“An Automated Technique for Improving Modal Test/Analysis Correlation,”** AIAA paper 82–0640, May, 1982.

Hasselman, T.K., **“Damping Synthesis from Substructure Tests,”** AIAA Journal, Vol. 14, No.10, October 1976, pp. 1409–1418.

Heylen, W., Sas, P., Leuven, K.U., **“Review of Model Optimization Techniques,”** Fifth IMAC, 1987, pp. 1177–1182.

Hurty, C.H., Rubinstein, M.F., **“Dynamics of Structures,”** Prentice–Hall, May 1965.

Gysin, H.P., **“Critical Application of The Error Matrix Method For Localization of the Finite Element Modeling Inaccuracies,”** Fourth IMAC Conf., 1986, pp. 1339–1351.

Ibrahim, S.R., Mikulcik, E.C., **“A Method For The Direct Identification Parameters From The Free Response,”** Shock And Vibration Bulletin, No. 47, September 1977, pp. 183–198.

Ibrahim, S.R., **“Random Decrement Technique for Identification of Structures,”** AIAA Journal of Spacecraft and Rockets, Vol.14, No.11, November 1977, pp. 696–700.

- Ibrahim, S.R., **"Modal Confidence Factor in Vibration Testing,"** AIAA Journal of Spacecraft and Rockets, Vol.15, September 1978, pp. 313–316.
- Ibrahim, S.R., **"Application of Random Time Domain Analysis to Dynamic Flight Measurement,"** Shock and Vibration Bulletin No. 49, Pt. 2, September 1979, pp. 165–170.
- Ibrahim, S.R., **"Correlation of Analysis and Test in Modeling of Structures: Assessment and Review,"** Fifth IMAC, 1987, pp. 1651– 1660.
- Ibrahim, S.R., **"Computation of Normal Modes from Identified Complex Modes,"** AIAA Journal, Vol.21, March 1983, pp. 446–451.
- Ibrahim, S.R., Mikulick, E.C., **"A Time Domain Modal Vibration Test Technique,"** Shock and Vibration Bulletin No. 43, Pt. 4, September 1979, pp. 165–170.
- Ibrahim, S.R., Mikulick, E.C., **"The Experimental Determination of Vibration Parameters from the Time Response,"** Shock and Vibration Bulletin No. 46, Pt. 5, September 1976, pp. 187–196.
- Ibrahim, S.R., Mikulick, E.C., **"A Method for the Direct Identification of Vibration Parameters from the Free Response,"** Shock and Vibration Bulletin No. 47, Pt. 4, September 1977, pp. 183–198.
- Ibrahim, S.R., Pappa R.S., **"Large Modal Survey Testing Using The Ibrahim Time Domain (ITD) Identification Technique,"** Journal of Spacecraft And Rockets, Vol. 19, September–October 1982, pp. 459–465.
- Imregun, M. and Visser, W.J. **"A Review of Model Updating Techniques".**
- Juang, J.N., Pappa, R.S., **"An Eigensystem Realization Algorithm For Modal Parameter Identification And Modal Reduction,"** Journal of Guidance, Control, and Dynamics, Vol. 8, No. 8, September October 1985, pp. 620–627.
- Kana, D.D., **"Substructure Energy Method for Prediction of Space Shuttle Modal Damping,"** Journal of Spacecraft, Vol. 12, No. 5, May 1975, pp. 294–301.
- Kaufman, L., **"The LZ-Algorithm To Solve The Generalized Eigenvalue Problem,"** SIAM Journal on Numerical Analysis, October 1974, pp. 997–1023.

- Kaufman, L., "Algorithm 496, The LZ Algorithm To Solve the Generalized Eigenvalue Problem for Complex Matrices,". SIAM Journal on Numerical Analysis, September 1975, pp. 271–281.
- Kim, Kook, Anderson, W.J., Sandstorm, R.E., "Nonlinear inverse Perturbation Method in Dynamic Analysis," AIAA Journal, Vol.21, September 1983, pp. 1310–1316.
- Kidder, R.L., "Reduction of Structural Frequency Equation," AIAA Journal, Vol. 11, April 1980, pp. 892.
- Komatsu, K., Sano, M., Kai, T., "Experimental Modal Analysis for Dynamic Models of Spacecraft," AIAA Journal, Vol. 14, May 1991, pp. 686–688.
- Lee, L.T., Hasselman, T.K., "Dynamic Model Verification of Large Structural Systems," SAE Paper 781047, November 1978.
- Leuridan, J.M., Kundrat, J., "Advanced Matrix Methods For Experimental Modal Analysis – A Multi-Matrix Method For Parameter Excitation,". First IMAC, November 1982, pp. 192–200.
- Link, M., "Identification of Physical System Matrices Using Incomplete Vibration Test Data,". Fourth IMAC, 1986, pp. 386–393.
- Maia, N.M.M., "An Introduction to the Singular Value Decomposition Technique (SVD)," IMAC, 1989., pp. 335–339.
- Marek, E.L., Branstetter, L.J., "Modal Survey and Test–Analysis Correlation of a Multiply–Configured Three–Stage Booster,".
- McGraw, J., "Orthogonalization of Measured Modes and Calculation of Influence Coefficients," AIAA Journal, Vol.7, April 1969, pp. 774–776.
- Moler, C.B., Stewart, G.W., "An Algorithm For Generalized Matrix Eigenvalue Problems,". SIAM Journal on Numerical Analysis, April 1973, pp. 241–256.
- Meirovitch, L., "Analytical Methods in Vibrations," Macmillan Publishing Co., 1967.

Nelson, R., **"Simplified Calculation of Eigenvector Derivatives,"** AIAA Journal, Vol. 14, September 1976, pp. 1201–1205.

Niedbal, N., **"Analytical Determination of Real Normal Modes From Measured Complex Responses,"** AIAA Paper 84–0995, May 1984, pp. 292–295.

Niedbal, I.N., Klusowski, D.L., **"Updating A Finite Element Model By Means Of Normal Mode Parameters,"** International Modal Analysis Seminar, 1988., pp. 1–15.

Ojalvo, I.U., **"Contrasts in Two Classes of Structural Dynamic Correlation Procedures,"** Fourth IMAC Conf., Vol. 1, 1986, pp. 81– 87.

Pappa, R.S., Ibrahim, S.R., **"A Parametric Study of the Ibrahim Time Domain Modal Identification Algorithm,"** Shock and Vibration Bulletin No. 51, Pt. 3, May 1981, pp. 43–72.

Park, Y.S., Park, H.S., **"Weighted–Error–Matrix Application to Detect Stiffness Damage By Dynamic Characteristic Measurement,"** Journal of Modal Analysis, April 1988, pp. 101–107.

**"Procedures for Updating Dynamic Mathematical Models,"** Technical Report, Dornier Systems (1985).

Rodden, W. P., **"A Method for Deriving Structural Influence Coefficient from Ground Vibration Tests,"** AIAA Journal, Vol.5, May 1967, pp. 991–1000.

Sidhu, J., Ewins, D.J., **"Correlation of Finite Element and Modal Test Studies of A Practical Structure,"** Second IMAC Conf., 1984, pp. 756–762.

Singal R.K., Vinegron F.R., Steele, T., **"Comparison of Different Methods of Modal Test on Spacecraft Representative Structure,"** The International Journal of Analytical and Experimental Modal Analysis 6(3):175–187, July 1991.

Soni, M.L., **"Damping Synthesis for Flexible Space Structures Using Combined Experimental and Analytical Models,"** AIAA Paper 85–0779, 1985.

Stetson, K.A., Palma, G.E., **“Inversion of First–Order Perturbation Theory and Its Application to Structural Design,”** AIAA Journal, Vol. 14, April 1976, pp. 454–460.

Sutter, T.R., Camarada, C.J., Walsh, J.L., Adelman, H.M., **“Comparison of Several Methods for Calculating Vibration Mode Shape Derivative,”** AIAA Journal, Vol. 26, No. 12, December 1988, pp. 1506–1511.

Targoff, W.P., **“Orthogonality Check and Correction of Measured Modes,”** AIAA Journal, Vol.14, April 1976, pp. 164–167.

Thorne A.R., **“Derivation of Mass and Stiffness Matrices From Dynamic Test Data,”** AIAA Paper 72–346, April 1972, pp. 1–5.

Ujihara, B.H., Dosoky, M.M., Tong, E.T., **“Improving A Nastran Dynamic Model With Test Data Using Linwood,”** 10th Nastran User’s Colloquium NASA Conference Publication 2249, 1982, pp. 74–76.

Wada, B.K., Garba, J.A., Chen, J.C., **“Development and Correlation: Viking Orbiter Analytical Dynamic Model With Modal Test,”** NASA Technical Memorandum 33–690, June 1974.

Wada, B.K., Garba, J.A., Chen, J.C., **“Modal Test and Analysis Correlation,”** AIAA Paper 84–22619, June 1974.

Weaver, W., Johnston, P.R., **“Structural Dynamics by Finite Element,”** Prentice–Hall, 1987.

Wei, F., **“Stiffness Matrix Correction from Incomplete Test Data,”** AIAA Journal, Vol. 18, October 1980, pp. 1274–1275.

Zhang, Q., Lallement, G., **“A Complete Procedure For The Adjustment of A Mathematical Model From The Identified Complex Modes,”** Fifth IMAC, 1987, pp. 1183–1190.

Zhang, Q., Lallement, G., **“A Complete Procedure For The Adjustment of A Mathematical Model From The Identified Complex Modes,”** Fifth IMAC, 1987, pp. 1183–1190.

## **ACKNOWLEDGMENTS**

I would like to sincerely thank my advisor, Professor Paul Lin, for his continuous guidance, effort, and for all he has taught me in my years as a graduate research student. I also would like to thank my doctoral dissertation committee members (Professor J. Frater, Professor C. Chang, Professor Svestka, Professor Rashidi) for all they have taught me; without this knowledge my research study could not have been possible. I also would like to thank all the faculty members in the Mechanical Engineering and Civil Engineering Departments at Cleveland State University for their much appreciated positive contributions to twelve years of my student life.

## **DEDICATION**

I would like to dedicate this dissertation to the following people:

My father and mother (Mohammad and Sepas Armand) for giving me strength and warmth.

My wife and children (Adrienne, Sean, and Alexis) for their patience during the years of my research study.

My father and master instructor in TAE KWON DO (Grand Master John Kim) for enforcing the three elements for success: Dedication, Desire, and Discipline.

My previous manager, (Dan Gauntner) at NASA, Lewis Research Center, for believing in me, and for giving me self-confidence.

My instructor (Professor John Frater) for starting me in my engineering career.



# **APPENDIX A**

## **FORTRAN PROGRAM**

```

C*****
C
C
C*****
      INTEGER LDA,LDB,LDEVEC,N
      PARAMETER (N=72,NN=36,MM=5,LDA=N,LDB=N,LDEVEC=N)
C*****
C If IINPUT=1, this program will be a simulation/generator program.
C If IINPUT=0, this program will be a test correlation program.
C If IDAMP=1, this program will correct the damping matrix.
C If ISTIFF=1, this program will correct the stiffness matrix.
      PARAMETER (IINPUT=0,IDAMP=1,ISTIFF=0)
C*****
      PARAMETER (nm=500)
C*****
C If IREAD=1, this program will read a MSC/NASTRAN output4 file.
C If IFREQ=1, this program will use the CG eigenproblem routine.
      PARAMETER (IREAD=1,IFREQ=1)
C*****
      integer ierr,matz
      double precision ar1(nm,n),ai1(nm,n),wr1(n),wl1(n),
x          zr1(nm,n),zi1(nm,n),zr2(nm,n),zi2(nm,n),
x          fv1(n),fv2(n),fv3(n)
      COMPLEX*16 P,T6
C      DOUBLE PRECISION REAL,AIMAG,R,T5
      DOUBLE PRECISION R,T5
      REAL REAL,AIMAG
      INTEGER I,NOUT,NN1,NN2
      REAL AMACH,GPICG,PI,VAR
C      COMPLEX A(LDA,N),ALPHA(N),B(LDB,N),BETA(N),EVAL(N),EVEC(LDEVEC,N)
      COMPLEX BETAI(500),EVAL(500),EVEC(500,500),ACORG(500,500)
C      COMPLEX A(500,500),B(500,500),BETAI(500),EVAL(500),EVEC(500,500)
      REAL ALPHA(500),BETA(500),ZR(500,500),ZI(500,500),ALFI(500)
      REAL AR(500,500),AI(500,500)
      REAL BR(500,500),BI(500,500),ALFR(500)
      COMPLEX AW(500,500),AVEC(500,500),AVEC T(500,500)
      COMPLEX DUM19(500,500),DUM20(500,500),DUM21(500,500)
      COMPLEX DUM1(500,500),DUM2(500,500),DUM3(500,500),DUM4(500,500)
      COMPLEX DUM6(500,500),DUM7(500,500),DUM8(500,500),DUM9(500,500)
      COMPLEX DUM10(500,500),DUM11(500,500),DUM12(500,500)
      COMPLEX DUM13(500,500),DUM14(500,500),DUM15(500,500)
      COMPLEX DUM16(500,500),DUM17(500,500),DUM18(500,500)
      COMPLEX AKC(500,500),AMC(500,500),ACC(500,500)
      COMPLEX TW(500,500),TVEC(500,500),TVEC T(500,500)
      COMPLEX AVEC_D(500,500),AVEC_D T(500,500),AKORG(500,500)
      COMPLEX TVEC_D(500,500),TVEC_D T(500,500)
      COMPLEX EB(500,500),TB O(500,500),TB(500,500),CFAC(500,500)
      COMPLEX EA(500,500),TA(500,500),AKT(500,500),ACT(500,500)
      COMPLEX DUMM1,DUM5(500,500)
      REAL AK(500,500),AM(500,500),AC(500,500)
      REAL ITER(500),AA(500,500)
      COMPLEX T,U
      COMPLEX A(500,500),B(500,500)
      INTEGER NROW,NCOL,J,K,I1,I2,I3,I4,IFLAG
C      EXTERNAL AMACH,GPICG,GVCCG,UMACH
      CHARACTER MATRIX*80
      CHARACTER*12 DOC1

C
C*****

```

```

C
C*****
  IF(IINPUT .EQ. 1) DOC1='TR441.IN'
  IF(IINPUT .EQ. 0) DOC1='TR44.IN'
  OPEN(UNIT=1,FILE=DOC1,STATUS='OLD')

  OPEN(UNIT=3,FILE='TR121.OUT',STATUS='OLD')
  OPEN(UNIT=2,FILE='TR12.OUT',status='new')
  OPEN(UNIT=4,FILE='SCR1.OUT',status='new')

  IF(IINPUT .EQ. 1) THEN
    OPEN(UNIT=5,FILE='SCR2.OUT',status='new')
  ELSE
    ENDIF

C*****
C
C*****
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C                                                                    $$$
C                                                                    $$$
C                                                                    $$$
C      Variable Definition                                          $$$
C                                                                    $$$
C                                                                    $$$
C                                                                    $$$
C                                                                    $$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

C*****
C N=2*NN
C NN          number of degree of freedom
C MM          number of modes
C AK(NN,NN)   stiffness matrix
C AC(NN,NN)   damping matrix
C AM(NN,NN)   mass matrix
C AK(NN,NN)   complex form of stiffness matrix
C AC(NN,NN)   complex form of damping matrix
C AM(NN,NN)   complex form of mass matrix
C A(N,N)      A matrix
C B(N,N)      B matrix
C EVAL(N)     subroutine calculated eigenvalues
C EVEC(N,N)   subroutine calculated eigenvectors
C AW(MM,MM)   analytical frequencies
C AVEC(NN,MM) analytical eigenvectors
C AVEC T(MM,NN) transpose of analytical eigenvectors
C TW(I,I)     test frequencies
C TVEC(NN,MM) test eigenvectors
C TVEC T(MM,NN) transpose of test eigenvectors
C AVEC D      analytical velocity
C AVEC D T    transpose of analytical velocity
C TVEC D      test velocity
C TVEC D T    transpose of test velocity
C EB(MM,MM)   analytical orthogonality number
C TB O(MM,MM) old test orthogonality number
C TB(MM,MM)   test orthogonality number
C CFAC(MM,MM)

```

```

C*****

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C                                     $$$
C                                     $$$
C                                     $$$
C      In this section, The stiffness, mass, $$$
C      damping, test eigenvectors, and test $$$
C      eigenvalues are read. $$$
C                                     $$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C      IFLAG=0
C      READ(1,17)NROW
C      WRITE(2,17)NROW
C17      FORMAT(I8)
C      TYPE 2, 'TITLE=DELTA AKC *4.'
C*****
C The following two lines are for use in LZIT routine.
C      DO 2 I=1,N
2          ITER(I)=-1

C*****
C If iread=1, then the output4 format of MSC/NASTRAN is read
C      IF(IREAD.EQ.1) THEN
C          CALL RNAS(AK,AM,AC,NN)
C          DO 3 I=1,NN
C              DO 4 J=1,NN
C                  AKC(I,J)=AK(I,J)
C                  AMC(I,J)=AM(I,J)
C                  ACC(I,J)=AC(I,J)
C                  ACORG(I,J)=ACC(I,J)
C                  AKORG(I,J)=AKC(I,J)
4              CONTINUE
3          CONTINUE
C*****

C      ELSE

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Read mass matrix

C      NROW=NN
C      NCOL=MM
C      DO 18 I=1,NROW
C          DO 19 J=1,NCOL
C              READ(1,20)AM(I,J)
C              AMC(I,J)=AM(I,J)
C              WRITE(*,20)AM(I,J)
C              WRITE(*,27)AMC(I,J)
27          FORMAT(10X,2D20.6)
20          FORMAT(F10.6)
19          CONTINUE
18          CONTINUE

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Read stiffness matrix

```

```

      DO 21 I=1,NROW
      DO 22 J=1,NROW
      READ(1,20)AK(I,J)
      AKC(I,J)=AK(I,J)
      AKORG(I,J)=AKC(I,J)
      TYPE *,I,J,AKC(I,J)
C      WRITE(*,20)AK(I,J)
C      WRITE(2,20)AK(I,J)
22      CONTINUE
21      CONTINUE

```

#####

C Read damping matrix

```

      DO 23 I=1,NROW
      DO 24 J=1,NROW
      READ(1,20)AC(I,J)
      ACC(I,J)=AC(I,J)
      ACORG(I,J)=ACC(I,J)
C      WRITE(*,20)AC(I,J)
C      WRITE(2,20)AC(I,J)
24      CONTINUE
23      CONTINUE

```

ENDIF

#####

C If IINPUT is not zero then this run is an input generator, and the  
C following read section will be skipped.

IF(IINPUT .EQ. 1) GO TO 5

#####

C Read the test frequencies

```

      DO 30 I =1,MM
      READ(3,31)TW(I,I)
31      FORMAT(2E20.9)
C      WRITE(*,31)TW(I,I)
30      CONTINUE
      MATRIX='          TEST EIGENVALUES    TW'
      CALL WRTT(MATRIX,TW,MM,MM)

```

#####

C Read the test eigenvectors and eigenvelocites

```

      DO 40 I=1,MM
      DO 41 J=1,NN
      READ(3,42)TVEC_D(J,I)
42      FORMAT(2E20.9)
41      CONTINUE
40      CONTINUE

```

```

      DO 43 I=1,MM
      DO 44 J=1,NN

```

```

      READ(3,42)TVEC(J,I)
44      CONTINUE
43      CONTINUE

      MATRIX='          TEST EIGENVECTORS      TVEC'
      CALL WRTT(MATRIX,TVEC,NN,MM)
      MATRIX='          TEST EIGENVELOCITIES    TVEC_D'
      CALL WRTT(MATRIX,TVEC_D,NN,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C                                          $$$
C                                          $$$
C                                          $$$
C   In this section, A and B matrices are formed. $$$
C                                          $$$
C                                          $$$
C                                          $$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

5      REWIND(UNIT=4)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Initialize the A and B matrices.

```

```

      DO 16 I=1,N
        EVAL(I)=(0.,0.)
        DO 15 J=1,N
          EVEC(I,J)=(0.,0.)
          AR(I,J)=(0.,0.)
          BR(I,J)=(0.,0.)
          AI(I,J)=(0.,0.)
          BI(I,J)=(0.,0.)
          DUM19(I,J)=AKC(I,J)
15      CONTINUE
16      CONTINUE

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C                                          $$$
C                                          $$$
C                                          $$$
C   In this section, complex eigenvalues
C   and eigenvectors are extracted using $$$
C   the CG subroutine                      $$$
C                                          $$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

C*****

```

```

C   If IFREQ=1 use the CG subroutine for eigenproblem solution.

```

```

      IF(IFREQ .NE. 1) GO TO 3320

```

```

C*****

```

```

C In this section the A=AR1,AI1 is written into file 4. The solution

```

C of the problem is  $AX = \text{Lamda} * X$ . The A matrix here is  $-(B^{**}-1)*A$ .

C DUM4 matrix is the inverse of DUM1 or Mass matrix.

```
CALL MATINVC(DUM19,DUM20,NN,NR)
CALL MULT(DUM19,DUM20,AMC,NN,NN,NN)
CALL MULT(DUM21,DUM20,ACC,NN,NN,NN)
```

C DUM5 matrix is the A matrix for CG subroutine.

```
C      CALL MULT(DUM5,DUM1,AKC,NN,NN,NN)
```

```
      DO 3000 I=1,NN
      DO 3010 J=1,NN
      WRITE(4,3020)AR(I,J),AI(I,J)
3020    FORMAT(2E20.9)
3010    CONTINUE
3000    CONTINUE
```

```
      DO 3030 I=1,NN
      DO 3040 J=1,NN
      IF(I .EQ. J) THEN
      T=(1.,0.)
      ELSE
      T=(0.,0.)
      ENDIF
      WRITE(4,3020)T
3040    CONTINUE
3030    CONTINUE
```

```
      DO 3070 I=1,NN
      DO 3080 J=1,NN
      DUM19(I,J)=-DUM19(I,J)
      WRITE(4,3020)DUM19(I,J)
3080    CONTINUE
3070    CONTINUE
```

```
      DO 3090 I=1,NN
      DO 3100 J=1,NN
      DUM21(I,J)=-DUM21(I,J)
      WRITE(4,3020)DUM21(I,J)
3100    CONTINUE
3090    CONTINUE
```

```
REWIND(UNIT=4)
```

C\*\*\*\*\*

```
      DO 3200 I=1,NN
      DO 3200 J=1,NN
3200    READ(4,3210)AR1(I,J),AI1(I,J)
3210    FORMAT(2E20.9)

      DO 3230 I=1,NN
      DO 3230 J=1,NN
3230    READ(4,3210)AR1(I,J+NN),AI1(I,J+NN)
```

```

DO 3240 I=1,NN
DO 3240 J=1,NN
3240 READ(4,3210)AR1(I+NN,J),AI1(I+NN,J)

DO 3250 I=1,NN
DO 3250 J=1,NN
3250 READ(4,3210)AR1(I+NN,J+NN),AI1(I+NN,J+NN)

REWIND(UNIT=4)

matz=1
ierr=1.E-20
CALL cg(nm,n,ar1,ai1,wr1,wil,matz,zr1,zi1,fv1,fv2,fv3,ierr)

IF(IFLAG .EQ. 5) GO TO 3301
TYPE *, '      MODE      REAL      IMAGINARY'
TYPE *, '      *****      *****      *****'
3301 DO 3300 I=1,N
WRITE(4,3311)1./CMPLX(WR1(I),WI1(I))
3311 FORMAT(2E20.9)
IF(IFLAG .EQ. 5) GO TO 3300
IF(I.LT.10)WRITE(*,3312)I,1./CMPLX(WR1(I),WI1(I))
3312 FORMAT(5X,I5,3X,F15.4,3X,F15.4)
3300 CONTINUE

C ***** NORMALIZE SO THAT MODULUS OF LARGEST
C COMPONENT OF EACH VECTOR IS 1 *****
C DO 3259 K=1,5
3259 IJ1=1
IJ2=N
3275 DO 3260 J = IJ1,N
DO 3270 I = 1, N
IF(I .EQ. 1) T5 = DABS(ZR1(I,J))
R = DABS(ZR1(I,J))
C TYPE *,I,ZR1(I,J),ZI1(I,J)
IF (R .GE. T5) THEN
T5 = R
I5=I
ELSE
ENDIF
3270 CONTINUE
C TYPE *, ' FIRST TIME', I5,ZR1(I5,J),ZI1(I5,J)

DO 3280 I=1,N
T6=
X DCMLPX(ZR1(I,J),ZI1(I,J))/DCMLPX(ZR1(I5,J),ZI1(I5,J))
C ZR1(I,J)=REAL(
C X CMPLX(ZR1(I,J),ZI1(I,J))/CMPLX(ZR1(I5,J),ZI1(I5,J))
C ZI1(I,J)=AIMAG(
C X CMPLX(ZR1(I,J),ZI1(I,J))/CMPLX(ZR1(I5,J),ZI1(I5,J))
C T6=
C X CMPLX(ZR1(I,J),ZI1(I,J))
C IF(ABS(ZR1(I,J)) .GT. 1.) THEN
C IJ1 = IJ1 - 1
C GO TO 3275
C ELSE
C ENDIF
C TYPE *,I,ZR1(I,J),ZI1(I,J)

```





```

53      B(I,J)=AM(I,J)
      BR(I,J)=AM(I,J)

```

```

      DO 54 I=I2,N
      DO 54 J=I2,N
      B(I,J)=-AK(I-I2+1,J-I2+1)
54      BR(I,J)=-AK(I-I2+1,J-I2+1)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Since the definition of A and B matrix in SIML is
C different, that is, A=-B and B=A, the following
C equivalencing is performed.

```

```

59      DO 116 I=1,N
      DO 115 J=1,N
      AA(I,J)=-B(I,J)
C      WRITE(*,117)I,J,A(I,J),B(I,J)
C      WRITE(2,117)I,J,A(I,J),B(I,J)
117      FORMAT(2X,I2,2X,I2,2X,F15.6,2X,F10.6,5X,F15.6,2X,F10.6)
115      CONTINUE
116      CONTINUE

```

```

      IF(IFREQ .EQ. 1) GO TO 3320

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C      $$$
C      $$$
C      $$$
C      In this section, complex eigenvalues
C      and eigenvectors are extracted.
C      $$$
C      $$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

C DETERMINE THE COMPLEX EIGENVALUES AND EIGENVECTORS

```

```

C      CALL GVCCG(N,AA,LDA,A,LDB,ALPHA,BETA,EVEC,LDEVEC)
C      CALL LZHES(N,AA,N,A,N,EVEC,N,.TRUE.)
C      CALL LZIT(N,AA,N,A,N,EVEC,N,.TRUE.,ITER,ALPHA,BETA)

```

```

      EPS1 = 1.0E-20
      IERR = 100000000
      CALL CQZHES(NM,N,BR,BI,AR,AI,.TRUE.,ZR,ZI)
      CALL CQZVAL(NM,N,BR,BI,AR,AI,EPS1,ALFR,ALFI,BETA,
$.TRUE.,ZR,ZI,IERR)
      CALL CQZVEC(NM,N,BR,BI,AR,AI,ALFR,ALFI,BETA,ZR,ZI)

```

```

      DO 10 I=1,N
      IF (BETA(I) .NE. 0.0) THEN
      EVAL(I)=CMPLX(ALFR(I),ALFI(I))/BETA(I)
      DO 110 J=1,N
      EVEC(I,J)=CMPLX(ZR(I,J),ZI(I,J))
      WRITE(*,11)EVEC(I,J)
C
11

```



C This section is to provide simulated/test modes and mode shapes for the  
C analysis model.

```

      IF(IINPUT .EQ. 0) GO TO 3380
      DO 3390 I=1,MM
        WRITE(5,3400)AW(I,I)
        FORMAT(2E20.9)
3400    CONTINUE
3390

```

```

DO 3410 I=1,MM
  DO 3420 J=1,NN
    WRITE(5,3400)AVEC_D(J,I)
3420  CONTINUE
3410  CONTINUE

```

```

DO 3430 I=1,MM
DO 3440 J=1,NN
WRITE(5,3400)AVEC(J,I)
3440 CONTINUE
3430 CONTINUE

```

```
IF(IINPUT.EQ. 1) THEN
TYPE *, '***** RENAME SCR2.OUT TO TR121.OUT *****'
TYPE *, 'RENAME SCR2.OUT TO TR121.OUT'
TYPE *, 'RENAME SCR2.OUT TO TR121.OUT'
TYPE *, 'RENAME SCR2.OUT TO TR121.OUT'
TYPE *, 'RENAME SCR2.OUT TO TR121.OUT'
ELSE
ENDIF
IF(IINPUT.EQ. 1) GO TO 1000
```

\*\*\*\*\*

C WRITE THE EIGENVALUES

```
3380 MATRIX=' ANALYSIS EIGENVALUES AW'
```

```
IF(IFLAG.EQ.5)GO TO 149
```

```
IF (IF1 .EQ. 902) THEN
MATRIX='MASS OPT., EIGENVALUE BASED ON C=(C-DELTA C)'
ELSE
```

```

      IF (IF1.EQ. 903) THEN
        MATRIX='MASS OPT., EIGENVALUE BASED ON C=(C-REAL(DELTA C))'
      ELSE

```

```

      ELSE
      IF (IF1 .EQ. 904) THEN
      MATRIX='STIFFNESS OPT., EIGENVALUE BASED ON C=(C-DELTA C)'
      ELSE
      ENDIF
    ENDIF
  ENDIF
  CALL WRTT (MATRIX,AW,MM,MM)

```

[illegible]













```

ENDIF
IF(NN .NE. MM) GO TO 1000

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$
C$$$$$$$$$          Direct Method      $$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

C If the AVEC matrix is not square, the following section is  
 C skipped, since the true inverse of AVEC and AVEC\_T can not  
 C be obtained in the following section.

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = M * AVEC_D

```

```

      CALL ZER(DUM1,NN,NN)
      CALL ZER(DUM2,NN,NN)
      CALL ZER(DUM3,NN,NN)
      CALL ZER(DUM4,NN,NN)
      CALL ZER(DUM5,NN,NN)
      CALL ZER(DUM6,NN,NN)
      CALL ZER(DUM7,NN,NN)

```

```

      CALL MULT(DUM1,AMC,AVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * AVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM2 = AVEC_T * DUM1 = AVEC_T * M * AVEC_D

```

```

      CALL MULT(DUM2,AVEC_T,DUM1,MM,NN,MM)
      MATRIX=' DUM2 = AVEC_T * M * AVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = M * AVEC

```

```

      CALL MULT(DUM1,AMC,AVEC,NN,NN,MM)
      MATRIX='      DUM1 = M * AVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM3 = AVEC_D_T * DUM1 = AVEC_D_T * M * AVEC

```

```

      CALL MULT(DUM3,AVEC_D_T,DUM1,MM,NN,MM)
      MATRIX=' DUM3 = AVEC_D_T * M * AVEC'
C      CALL WRTT(MATRIX,DUM3,MM,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE AVEC_T*C*AVEC=EA-DUM2-DUM3

```



```

      CALL ZER(DUM3,NN,NN)
      CALL ZER(DUM4,NN,NN)
      CALL ZER(DUM5,NN,NN)
      CALL ZER(DUM6,NN,NN)
      CALL ZER(DUM7,NN,NN)
      CALL ZER(DUM8,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC_D

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_D_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM3 = EB + DUM1 = EB + TVEC_D_T * M * TVEC_D

      CALL ADD(DUM3,EB,DUM2,MM,MM,MM)
      MATRIX='      DUM3 = EB + TVEC_D_T * M * TVEC_D'
      CALL WRTT(MATRIX,DUM3,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = 1./DUM3 = (EB + TVEC_D_T * M * TVEC_D)**-1

      CALL MATINVC(DUM3,DUM4,MM,MM)
      MATRIX='      DUM4 = (EB + TVEC_D_T * M * TVEC_D)**-1'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM5 = DUM4*TVEC_T =
C      ((EB + TVEC_D_T * M * TVEC_D)**-1)TVEC_T

      CALL MULT(DUM5,DUM4,TVEC,MM,MM,NN)
      MATRIX='DUM5=((EB + TVEC_D_T * M * TVEC_D)**-1)TVEC_T'
C      CALL WRTT(MATRIX,DUM5,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE (K*)**-1 = DUM6 = TVEC*DUM5 =
C      TVEC((EB + TVEC_D_T * M * TVEC_D)**-1)TVEC_T

      CALL MULT(DUM6,TVEC,DUM5,NN,MM,NN)
      MATRIX='(K*)**-1=TVEC((EB + TVEC_D_T * M * TVEC_D)**-1)TVEC_T'
C      CALL WRTT(MATRIX,DUM6,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine the inverse of K matrix. DUM2=(K)**-1

      DO 2700 I=1,NN
      DO 2701 J=1,NN
      DUM1(I,J)=AKC(I,J)
2701 CONTINUE
2700 CONTINUE

```

```

      CALL MATINVC(DUM1,DUM2,NN,NR)
      MATRIX=' DUM2 = (K)**-1'
      CALL WRTT(MATRIX,DUM2,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM4 = DUM2 - DUM6 = ((K)**-1) - ((K*)** -1)

      CALL SUBTT(DUM4,DUM2,DUM6,NN,NN)
      MATRIX=' DUM4 = ((K)**-1) - ((K*)** -1)'
C      CALL WRTT(MATRIX,DUM4,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM5 = DUM4 * K = (((K)**-1) - ((K*)** -1))K

      CALL MULT(DUM5,DUM4,AKC,NN,NN,NN)
      MATRIX=' DUM4 = (((K)**-1) - ((K*)** -1))K'
C      CALL WRTT(MATRIX,DUM5,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6 = K * DUM5 = K(((K)**-1) - ((K*)** -1))K

      CALL MULT(DUM6,AKC,DUM5,NN,NN,NN)
      CALL SUBTT(DUM7,AKC,DUM4,NN,NN)
      MATRIX=' DUM4 = K(((K)**-1) - ((K*)** -1))K'
      CALL WRTT(MATRIX,DUM6,NN,NN)
      MATRIX=' DUM7 = ERROR MATRIX'
      CALL WRTT(MATRIX,DUM7,NN,NN)

```

```

C If idamp is not 1, then correction of damping is not
C desired.

```

```

      IF(IDAMP .NE. 1) GO TO 8

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$          DAMPING CORRECTION          $$$$$$$$$$$$
C$$$$$$$$$$$      Mass Optimization Method      $$$$$$$$$$$$
C$$$$$$$$$$$          $$$$$$$$$$$$
C$$$$$$$$$$$          $$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

C Initialize the dummy matrices.

```

```

9      CALL ZER(DUM1,NN,NN)
      CALL ZER(DUM2,NN,NN)
      CALL ZER(DUM3,NN,NN)
      CALL ZER(DUM4,NN,NN)
      CALL ZER(DUM5,NN,NN)
      CALL ZER(DUM6,NN,NN)
      CALL ZER(DUM7,NN,NN)
      CALL ZER(DUM8,NN,NN)

```

```

CALL ZER(DUM9,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE M * TVEC_D

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_D_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE K * TVEC

      CALL MULT(DUM1,AKC,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = K * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM3 = TVEC_T * DUM1 = TVEC_T * K * TVEC

      CALL MULT(DUM3,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM3 = TVEC_T * K * TVEC'
C      CALL WRTT(MATRIX,DUM3,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TB = DUM3 - DUM2

      CALL SUBTT(TB,DUM3,DUM2,MM,MM)
      MATRIX='TB = TVEC_T * K * TVEC - TVEC_D_T * M * TVEC_D'
      CALL WRTT(MATRIX,TB,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TA = TB / -TW
      DO 205 I=1,MM
      DO 206 J=1,MM
      TA(J,I)=TB(J,I)/-TW(I,I)
206      CONTINUE
205      CONTINUE
      MATRIX='      TA = TB / -TW'
      CALL WRTT(MATRIX,TA,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Calculate the mess inside the paranteses

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = M * TVEC_D

      CALL ZER(DUM1,NN,NN)
      CALL ZER(DUM2,NN,NN)
      CALL ZER(DUM3,NN,NN)
      CALL ZER(DUM4,NN,NN)

```







```

      CALL MULT(DUM5,DUM1,DUM4,MM,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6=DUM4*DUM5
C   DUM6=((TVEC_T*M*TVEC)**-1)*(mess)*(TVEC_T*M*TVEC)**-1

      CALL MULT(DUM6,DUM4,DUM5,MM,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM8=DUM6*DUM7
C DUM8=((TVEC_T*M*TVEC)**-1)*(mess)*((TVEC_T*M*TVEC)**-1)*TVEC_T*M

      CALL MULT(DUM8,DUM6,DUM7,MM,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6=DUM2*DUM8
C DUM6=M*TVEC*
C   ((TVEC_T*M*TVEC)**-1)*
C   (mess)*
C   ((TVEC_T*M*TVEC)**-1)*
C   TVEC_T*M

      CALL MULT(DUM6,DUM2,DUM8,NN,MM,NN)
      MATRIX='          MASS OPT. DELTA C'
      CALL WRTT(MATRIX,DUM6,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine the new C matrices.

      DO 306 I=1,NN
      DO 307 J=1,NN
        DUM1(I,J)=ACC(I,J)
        ACORG(I,J)=ACC(I,J)
        DUM2(I,J)=(0.,0.)
        DUMM=REAL(DUM6(I,J))
        DUM2(I,J)=CMPLX(DUMM,0.)
307    CONTINUE
306    CONTINUE

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C If Iflag is 4 then the new eigenproblem with the new
C C matrix is solved.

      IFLAG = 4

C The strategy will be that one time delta C will be added,
C and one time it will be subtracted from the C matrix.

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Add the delta C to C matrix and solve eigenproblem.

      IF1 = 901
901    IF(IF1.GT. 901)GO TO 902
      CALL ADD(DUM3,DUM1,DUM6,NN,NN)
      CALL SUBTT(DUM3,DUM1,DUM6,NN,NN)
      MATRIX='          DELTA C ADDED TO THE C MATRIX, NEW C'
      CALL WRTT(MATRIX,DUM3,NN,NN)
      DO 303 I=1,NN

```



```

      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_D_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = K * TVEC

      CALL MULT(DUM1,AKC,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = K * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_T * DUM1 = TVEC_T * K * TVEC

      CALL MULT(DUM4,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_T * K * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TB = DUM4 - DUM2

      CALL SUBTT(TB,DUM4,DUM2,MM,MM)
      MATRIX='TB = TVEC_T * K * TVEC - TVEC_D_T * M * TVEC_D'
      CALL WRTT(MATRIX,TB,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * AVEC_D

      CALL MULT(DUM1,AMC,AVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * AVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = AVEC_T * DUM1 = AVEC_T * M * AVEC_D

      CALL MULT(DUM2,AVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = AVEC_T * M * AVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * AVEC

      CALL MULT(DUM1,AMC,AVEC,NN,NN,MM)
      MATRIX='      DUM1 = M * AVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = AVEC_D_T * DUM1 = AVEC_D_T * M * AVEC

      CALL MULT(DUM4,AVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = AVEC_D_T * M * AVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

C DETERMINE DUM5 = DUM2 + DUM4 =
C      AVEC_T * M * AVEC_D + AVEC_D_T * M * AVEC

      CALL ADD(DUM5,DUM2,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = C * AVEC

      CALL MULT(DUM1,DUM3,AVEC,NN,NN,MM)
      MATRIX='      DUM1 = C * AVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = AVEC_T * DUM1 = AVEC_T * C * AVEC

      CALL MULT(DUM4,AVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = AVEC_T * C * AVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE EA = DUM4 + DUM5

      CALL ADD(EA,DUM4,DUM2,MM,MM)
      WRITE(2,800)
800  FORMAT(1X,'ORTHO BASED ON IMPROVED MATRICES AND VECTORS')
      MATRIX='EA=AVEC_D T*C*AVEC+AVEC_D_T*M*AVEC_D+AVEC_T*M*AVEC_D_T'
      CALL WRTT(MATRIX,EA,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC_D

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_T * DUM1 = TVEC_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC

      CALL MULT(DUM1,AMC,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC

      CALL MULT(DUM4,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_D_T * M * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM5 = DUM2 + DUM4 =
C      TVEC_T * M * TVEC_D + TVEC_D_T * M * TVEC

```

```

      CALL ADD(DUM5,DUM2,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = C * TVEC

      CALL MULT(DUM1,DUM3,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = C * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_T * DUM1 = TVEC_T * C * TVEC

      CALL MULT(DUM4,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_T * C * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TA = DUM4 + DUM5

      CALL ADD(TA,DUM4,DUM2,MM,MM)
      WRITE(2,801)
801  FORMAT(1X,'ORTHO BASED ON TEST VECTORS AND IMPROVED MATRICES')
      MATRIX='TA=TVEC_D T*C*TVEC+TVEC_D_T*M*TVEC_D+TVEC_T*M*TVEC_D_T'
      CALL WRTT(MATRIX,TA,MM,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC_D

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_T * DUM1 = TVEC_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC

      CALL MULT(DUM1,AMC,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC

      CALL MULT(DUM4,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_D_T * M * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM5 = DUM2 + DUM4 =
C      TVEC_T * M * TVEC_D + TVEC_D_T * M * TVEC

      CALL ADD(DUM5,DUM2,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = C * TVEC

```



\$

\$

C Initialize the dummy matrices.

```
CALL ZER(DUM1,NN,NN)
CALL ZER(DUM2,NN,NN)
CALL ZER(DUM3,NN,NN)
CALL ZER(DUM4,NN,NN)
CALL ZER(DUM5,NN,NN)
CALL ZER(DUM6,NN,NN)
CALL ZER(DUM7,NN,NN)
CALL ZER(DUM8,NN,NN)
```

\$

C Determine DUM2 = K \* TVEC

```
903 CALL MULT(DUM2,AKC,TVEC,NN,NN,MM)
    MATRIX='      DUM2 = K * TVEC'
C    CALL WRTT(MATRIX,DUM2,NN,MM)
```

\$

C Determine DUM7 = TVEC\_T \* K

```
CALL MULT(DUM7,TVEC_T,AKC,MM,NN,NN)
MATRIX='      DUM7 = TVEC_T * K'
C    CALL WRTT(MATRIX,DUM7,NN,MM)
```

\$

C Determine DUM3 = TVEC\_T \* DUM2 = TVEC\_T \* K \* TVEC

```
CALL MULT(DUM3,TVEC_T,DUM2,MM,NN,MM)
MATRIX=' DUM3 = TVEC_T * K * TVEC'
C    CALL WRTT(MATRIX,DUM3,MM,MM)
```

\$

C Determine DUM4 = (TVEC\_T \* K \* TVEC)\*\*-1

```
DO 260 I=1,MM
  DO 261 J=1,MM
    DUM6(I,J)=DUM3(I,J)
261 CONTINUE
260 CONTINUE
```

```
CALL MATINVC(DUM3,DUM4,MM,MM)
MATRIX=' DUM4 = (TVEC_T * K * TVEC)**-1'
C    CALL WRTT(MATRIX,DUM4,MM,MM)
```

\$

C Determine DUM5=DUM9\*DUM4=(mess)\*(TVEC\_T\*K\*TVEC)\*\*-1

```
CALL MULT(DUM5,DUM9,DUM4,MM,MM,MM)
```

\$

C Determine DUM6=DUM4\*DUM5

```
C    DUM6=((TVEC_T*K*TVEC)**-1)*(mess)*(TVEC_T*K*TVEC)**-1
```

```

      CALL MULT(DUM6,DUM4,DUM5,MM,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM8=DUM6*DUM7
C DUM8=((TVEC_T*K*TVEC)**-1)*(mess)*((TVEC_T*K*TVEC)**-1)*TVEC_T*K

      CALL MULT(DUM8,DUM6,DUM7,MM,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6=DUM2*DUM8
C DUM6=K*TVEC*
C      ((TVEC_T*K*TVEC)**-1)*
C      (mess)*
C      ((TVEC_T*K*TVEC)**-1)*
C      TVEC_T*K

      CALL MULT(DUM6,DUM2,DUM8,NN,MM,NN)
      MATRIX='          STIFFNESS OPT. DELTA C'
      CALL WRTT(MATRIX,DUM6,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine the new C matrices.

      DO 262 I=1,NN
      DO 263 J=1,NN
      DUM1(I,J)=ACORG(I,J)
      ACC(I,J)=ACORG(I,J)
      DUM2(I,J)=(0.,0.)
      DUMM=REAL(DUM6(I,J))
      DUM2(I,J)=CMPLX(DUMM,0.)
263      CONTINUE
262      CONTINUE

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Subtract the delta C from C matrix and solve eigenproblem.

      IF(IF1 .GE. 904) GO TO 1000
      CALL SUBTT(DUM3,DUM1,DUM6,NN,NN)
      MATRIX='          DELTA C BASED ON STIFFNESS'
      CALL WRTT(MATRIX,DUM3,NN,NN)
      DO 209 I=1,NN
      DO 210 J=1,NN
      ACC(I,J)=DUM3(I,J)
      AC(I,J)=DUM3(I,J)
210      CONTINUE
209      CONTINUE
      IF1 = IF1 + 1
      IF(IFLAG .EQ. 4) GO TO 5

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C$$$$$$$$$          STIFFNESS AND DAMPING CORRECTION          $$$$$$$$$$
C$$$$$$$$$          Optimization Method                        $$$$$$$$$$
C$$$$$$$$$          $$$$$$$$$$                                $$$$$$$$$$
C$$$$$$$$$

```





```

C Determine DUM10 = TVEC_T * M**2
      CALL MULT(DUM10,TVEC_T,DUM14,MM,NN,NN)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM11 = M**2 * TVEC_T
      CALL MULT(DUM11,DUM14,TVEC,NN,NN,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = M * TVEC_PT
      CALL MULT(DUM1,AMC,DUM18,NN,NN,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM5 = TVEC_T * DUM1 = TVEC_T * M * TVEC_PT
      CALL MULT(DUM5,TVEC_T,DUM1,MM,NN,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM12 = DUM5**(-1) = (TVEC_T * M * TVEC_PT)**(-1)
      CALL MATINVC(DUM5,DUM12,MM,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM16 = (TVEC_T * M * TVEC_PT)**(-1) * TVEC_T * M**2
      CALL MULT(DUM16,DUM12,DUM10,MM,MM,NN)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM17 = M**2 * TVEC * (TVEC_T * M * TVEC_PT)**(-1)
      CALL MULT(DUM17,DUM11,DUM12,NN,MM,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM5 = DUM2 + DUM3 =
C      (TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D)
      CALL ADD(DUM5,DUM2,DUM3,MM,MM)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6 = DUM5 * DUM16 =
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D)) *
C (TVEC_T * M * TVEC_PT)**(-1) * TVEC_T * M**2
      CALL MULT(DUM6,DUM5,DUM16,MM,MM,NN)
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM7 = DUM17 * DUM6 =
C (M**2 * TVEC * (TVEC_T * M * TVEC_PT)**(-1) *
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D)) *
C (TVEC_T * M * TVEC_PT)**(-1) * TVEC_T * M**2)
      CALL MULT(DUM7,DUM17,DUM6,NN,MM,NN)
      MATRIX='      DUM7'
C      CALL WRTT(MATRIX,DUM7,NN,NN)
C      STOP
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Assume a value for the initial value of ACT

```

C Say 90% of the original.

```
DO I=1,NN
DO J=1,NN
DUM5(I,J)=ACORG(I,J)
ACT(I,J)=ACORG(I,J)
END DO
END DO
```

```
DUM5(3,3)=(.9,0.)
DUM5(6,6)=(.9,0.)
DUM5(9,9)=(.9,0.)
DUM5(15,15)=(.9,0.)
DUM5(18,18)=(.9,0.)
DUM5(21,21)=(.9,0.)
DUM5(27,27)=(.9,0.)
DUM5(30,30)=(.9,0.)
DUM5(33,33)=(.9,0.)
```

```
C ACT(2,2)=(.95,0.)
C ACT(4,4)=(.95,0.)
C ACT(5,5)=(.95,0.)
C ACT(6,6)=(.95,0.)
```

C the step size is .05. The assumed value of ACT can grow to  
C 105% by 3 steps, IJ=3.

```
DJ=.025
IJ=3
```

C IJI is counter for convergence.

```
IJI=0
```

```
IIJII=1
TMAG1=1.
TMAG2=1.
```

C\$  
C The following are the elements of C matrix.  
C as they grow.

```
IIJ2=-1
DO II2=1,IJ
IIJ2=IIJ2+1
ACT(3,3)=DUM5(3,3)+IIJ2*DJ
IJ4=-1
DO II4=1,IJ
IJ4=IJ4+1
ACT(6,6)=DUM5(6,6)+IJ4*DJ
IJ5=-1
DO II5=1,IJ
IJ5=IJ5+1
ACT(9,9)=DUM5(9,9)+IJ5*DJ
IJ6=-1
DO II6=1,IJ
IJ6=IJ6+1
ACT(15,15)=DUM5(15,15)+IJ6*DJ
IJ8=-1
DO II8=1,IJ
```

```

      IJ8=IJ8+1
      ACT(18,18)=DUM5(18,18)+IJ8*DJ
      IJ10=-1
      DO II10=1,IJ
        IJ10=IJ10+1
        ACT(21,21)=DUM5(21,21)+IJ10*DJ
        IJ11=-1
        DO II11=1,IJ
          IJ11=IJ11+1
          ACT(27,27)=DUM5(27,27)+IJ11*DJ
          IJ12=-1
          DO II12=1,IJ
            IJ12=IJ12+1
            ACT(30,30)=DUM5(30,30)+IJ12*DJ
            IJ33=-1
            DO II33=1,IJ
              IJ33=IJ33+1
              ACT(33,33)=DUM5(33,33)+IJ33*DJ

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = ACT * TVEC

      CALL MULT(DUM1,ACT,TVEC,NN,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM4 = TVEC_T * DUM1 = TVEC_T * ACT * TVEC

      CALL MULT(DUM4,TVEC_T,DUM1,MM,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM8 = DUM4 * DUM16 =
C (TVEC_T * ACT * TVEC)*
C ((TVEC_T * M * TVEC_PT)**-1 * TVEC_T * M**2)

      CALL MULT(DUM8,DUM4,DUM16,MM,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM9 = DUM17 * DUM8 =
C (M**2 * TVEC *(TVEC_T * M * TVEC_PT)**-1)
C (TVEC_T * ACT * TVEC)*
C ((TVEC_T * M * TVEC_PT)**-1 * TVEC_T * M**2)

      CALL MULT(DUM9,DUM17,DUM8,NN,MM,NN)
      MATRIX='      DUM9'
C      CALL WRTT(MATRIX,DUM9,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1
C (M**2 * TVEC *(TVEC_T * M * TVEC_PT)**-1)*
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1 * TVEC_T * M**2

      CALL ADD(DUM1,DUM7,DUM9,NN,NN)
      MATRIX='      DUM1'
C      CALL WRTT(MATRIX,DUM1,NN,NN)
C
C      STOP

      IJI=IJI+1

```

```

C          TT5=0.
C          RR=0.
C          DO J=1,NN
C            DO I=1,NN
C              IF(I .EQ. 1.AND.J .EQ. 1)TT5=ABS(REAL(DUM1(I,J)))
C              RR=ABS(REAL(DUM1(I,J)))
C              IF(RR .GT. TT5) THEN
C                TT5=RR
C              ELSE
C                ENDIF
C            END DO
C          END DO

C          CALL ZER(ACT,NN,NN)
C          CALL ADD(ACT,ACORG,DUM1,NN,NN)
C          MATRIX='      DUM2'
C          CALL WRTT(MATRIX,DUM2,NN,NN)

          DO 2556 I=1,NN
            DO 2557 J=1,NN
              ACC(I,J)=ACT(I,J)
2557       CONTINUE
2556       CONTINUE

C          IF(TT5. LT. 0.001) GO TO 2559

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = K * DUM18 = K * TVEC_PT

          CALL MULT(DUM1,AKORG,DUM18,NN,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM2 = TVEC_T * DUM1 = TVEC_T * K * TVEC_PT

          CALL MULT(DUM2,TVEC_T,DUM1,MM,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = M * TVEC_D

          CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM3 = TVEC_T * DUM1 = TVEC_T * M * TVEC_D

          CALL MULT(DUM3,TVEC_T,DUM1,MM,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = ACT * TVEC

          CALL MULT(DUM1,ACT,TVEC,NN,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM4 = TVEC_T * DUM1 = TVEC_T * ACT * TVEC

          CALL MULT(DUM4,TVEC_T,DUM1,MM,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM6 = DUM2 + DUM3 =
C (TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D)

```

```

CALL ADD(DUM6,DUM2,DUM3,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM8 = DUM6 + DUM4 =
C (TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC)

CALL ADD(DUM8,DUM6,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM14 = DUM8 * DUM12
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1

CALL MULT(DUM14,DUM8,DUM12,MM,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine LAMDA_T = DUM15 = DUM12 * DUM14
C (TVEC_T * M * TVEC_PT)**-1 *
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1

CALL MULT(DUM15,DUM12,DUM14,MM,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM1 = TVEC_T * M

CALL MULT(DUM1,TVEC_T,AKC,MM,NN,NN)
C CALL MULT(DUM1,TVEC_T,AKC,MM,NN,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM2 = M * TVEC_PT

CALL MULT(DUM2,AKC,DUM18,NN,NN,MM)
C CALL MULT(DUM2,AKC,DUM18,NN,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM3 = LAMDA_T * DUM1
C ((TVEC_T * M * TVEC_PT)**-1 *
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1)*
C (TVEC_T * M)

CALL MULT(DUM3,DUM15,DUM1,MM,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine DUM4 = DUM2 * DUM3 = DUM2 * LAMDA_T * DUM1
C (M * TVEC_PT)*
C ((TVEC_T * M * TVEC_PT)**-1 *
C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1)*
C (TVEC_T * M)

CALL MULT(DUM4,DUM2,DUM3,NN,MM,NN)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C Determine AKT = NEW VALUE OF AKC = AKC + DUM4
C AKC +
C (M * TVEC_PT)*
C ((TVEC_T * M * TVEC_PT)**-1 *

```

```

C ((TVEC_T * K * TVEC) + (TVEC_T * M * TVEC_D) + (TVEC_T * ACT * TVEC))*
C (TVEC_T * M * TVEC_PT)**-1)*
C (TVEC_T * M)

```

```

      DO IK=1,NN
        DO JK=1,NN
          DUM4(IK,JK)=1.0*DUM4(IK,JK)
        END DO
      END DO

```

```

      CALL SUBTT(AKT,AKORG,DUM4,NN,NN)
C      CALL ADD(AKT,AKORG,DUM4,NN,NN)

```

```

      DO IK=1,NN
        DO JK=1,NN
          AKC(IK,JK)=AKT(IK,JK)
        END DO
      END DO

```

```

      IFLAG = 5

```

```

      CALL MULT(DUM1,AMC,AVEC_D,NN,NN,MM)
      CALL MULT(DUM2,AVEC_D_T,DUM1,MM,NN,MM)

```

```

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      CALL MULT(DUM3,TVEC_D_T,DUM1,MM,NN,MM)

```

```

      DO IK=1,MM
        DUM1(IK,IK)=DUM2(IK,IK)/DUM3(IK,IK)
      END DO

```

```

      TT21=ABS(REAL(DUM1(1,1)))
      TT22=ABS(REAL(DUM1(2,2)))
      TT23=ABS(REAL(DUM1(3,3)))
      TT24=ABS(REAL(DUM1(4,4)))

```

```

      TT1=ABS(REAL(AW(1,1)))
      TT2=ABS(REAL(TW(1,1)))

```

```

      TT5=ABS(ABS(REAL(AW(2,2)))-ABS(REAL(TW(2,2))))
      TT6=ABS(ABS(REAL(AW(1,1)))-ABS(REAL(TW(1,1))))
      TT7=ABS(ABS(AIMAG(AW(2,2)))-ABS(AIMAG(TW(2,2))))
      TT8=ABS(ABS(AIMAG(AW(1,1)))-ABS(AIMAG(TW(1,1))))
      TT55=ABS(ABS(REAL(AW(3,3)))-ABS(REAL(TW(3,3))))
      TT66=ABS(ABS(REAL(AW(4,4)))-ABS(REAL(TW(4,4))))
      TT9=REAL(AVEC_D(1,2))
      TT10=REAL(AVEC_D(1,3))
      TT11=REAL(AVEC_D(1,18))
      TT12=REAL(AVEC_D(1,25))
      TT13=REAL(AVEC_D(2,2))
      TT14=REAL(AVEC_D(2,3))
      TT15=REAL(AVEC_D(2,18))
      TT16=REAL(AVEC_D(2,25))

```

```

      IF(TT5 .LT. 1. .AND. TT6 .LT. 1.) THEN
      IF(TT7 .LT. 1. .AND. TT8 .LT. 1.) THEN
      IF(TT55 .LT. 1. .AND. TT66 .LT. 1.) THEN

```

```

C      IF(TT21 .LT. 1.004 .AND. TT21 .GT. 0.996) THEN
C      IF(TT22 .LT. 1.004 .AND. TT22 .GT. 0.996) THEN
      IF(TT21 .LT. 1.02 .AND. TT21 .GT. 0.98) THEN

```





```

ELSE
ENDIF

IF(IFLAG.EQ. 5) GO TO 5
2558 TYPE *,IJI

998         END DO
            END DO
            END DO
            END DO
            END DO
            END DO
            END DO
            END DO
            END DO

C*****
C DETERMINE THE ORTHOGONALITY CONDITION.

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * AVEC_D

999     CALL MULT(DUM1,AMC,AVEC_D,NN,NN,MM)
        MATRIX='      DUM1 = M * AVEC_D'
C       CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = AVEC_D_T * DUM1 = AVEC_D_T * M * AVEC_D

        CALL MULT(DUM2,AVEC_D_T,DUM1,MM,NN,MM)
        MATRIX='      DUM2 = AVEC_D_T * M * AVEC_D'
C       CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = K * AVEC

        CALL MULT(DUM1,AKT,AVEC,NN,NN,MM)
        MATRIX='      DUM1 = K * AVEC'
C       CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = AVEC_T * DUM1 = AVEC_T * K * AVEC

        CALL MULT(DUM4,AVEC_T,DUM1,MM,NN,MM)
        MATRIX='      DUM4 = AVEC_T * K * AVEC'
C       CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE EB = DUM4 - DUM2

        CALL SUBTT(EB,DUM4,DUM2,MM,MM)
        WRITE(2,800)
        MATRIX='EB = AVEC_T * K * AVEC - AVEC_D_T * M * AVEC_D'
        CALL WRTT(MATRIX,EB,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```



```

      MATRIX='          DUM4 = TVEC T * K * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TB = DUM4 - DUM2

      CALL SUBTT(TB,DUM4,DUM2,MM,MM)
      WRITE(2,802)
      MATRIX='TB = TVEC T * K * TVEC - TVEC_D_T * M * TVEC_D'
      CALL WRTT(MATRIX,TB,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * AVEC_D

      CALL MULT(DUM1,AMC,AVEC_D,NN,NN,MM)
      MATRIX='          DUM1 = M * AVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = AVEC_T * DUM1 = AVEC_T * M * AVEC_D

      CALL MULT(DUM2,AVEC_T,DUM1,MM,NN,MM)
      MATRIX='          DUM2 = AVEC T * M * AVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * AVEC

      CALL MULT(DUM1,AMC,AVEC,NN,NN,MM)
      MATRIX='          DUM1 = M * AVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = AVEC_D_T * DUM1 = AVEC_D_T * M * AVEC

      CALL MULT(DUM4,AVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='          DUM4 = AVEC_D T * M * AVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM5 = DUM2 + DUM4 =
C      AVEC_T * M * AVEC_D + AVEC_D_T * M * AVEC

      CALL ADD(DUM5,DUM2,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = C * AVEC

      CALL MULT(DUM1,ACT,AVEC,NN,NN,MM)
      MATRIX='          DUM1 = C * AVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = AVEC_T * DUM1 = AVEC_T * C * AVEC

      CALL MULT(DUM4,AVEC_T,DUM1,MM,NN,MM)
      MATRIX='          DUM4 = AVEC T * C * AVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```



```

      WRITE(2,801)
C801  FORMAT(1X,'ORTHO BASED ON TEST VECTORS AND IMPROVED MATRICES')
      MATRIX='TA=TVEC D T*C*TVEC+TVEC_D_T*M*TVEC_D+TVEC_T*M*TVEC_D_T'
      CALL WRTT(MATRIX,TA,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC_D

      CALL MULT(DUM1,AMC,TVEC_D,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC_D'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM2 = TVEC_T * DUM1 = TVEC_T * M * TVEC_D

      CALL MULT(DUM2,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM2 = TVEC_T * M * TVEC_D'
C      CALL WRTT(MATRIX,DUM2,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = M * TVEC

      CALL MULT(DUM1,AMC,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = M * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_D_T * DUM1 = TVEC_D_T * M * TVEC

      CALL MULT(DUM4,TVEC_D_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_D_T * M * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM5 = DUM2 + DUM4 =
C      TVEC_T * M * TVEC_D + TVEC_D_T * M * TVEC

      CALL ADD(DUM5,DUM2,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM1 = C * TVEC

      CALL MULT(DUM1,ACORG,TVEC,NN,NN,MM)
      MATRIX='      DUM1 = C * TVEC'
C      CALL WRTT(MATRIX,DUM1,NN,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE DUM4 = TVEC_T * DUM1 = TVEC_T * C * TVEC

      CALL MULT(DUM4,TVEC_T,DUM1,MM,NN,MM)
      MATRIX='      DUM4 = TVEC_T * C * TVEC'
C      CALL WRTT(MATRIX,DUM4,MM,MM)

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C DETERMINE TA = DUM4 + DUM5

      CALL ADD(TA,DUM4,DUM2,MM,MM)
      WRITE(2,802)
C802  FORMAT(1X,'ORTHO BASED ON TEST VECTORS AND ANALYTICAL MATRICES')

```

```

MATRIX='TA=TVEC D T*C*TVEC+TVEC_D_T*M*TVEC_D+TVEC_T*M*TVEC_D_T'
CALL WRTT(MATRIX,TA,MM,MM)

```

```

C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

```

```

1000      END

```

```

C=SUBROUTINE=ZER
      SUBROUTINE ZER(A,N,M)
C A(N,M)=0.
      COMPLEX A(500,500)
      INTEGER I,J,M,N
      DO 1 I=1,N
        DO 2 J=1,M
          A(I,J)=(0.,0.)
2      CONTINUE
1      CONTINUE
      RETURN
      END

```

```

C=SUBROUTINE=WRT
      SUBROUTINE WRT(A,N,M)
      COMPLEX A(N,M)
      INTEGER I,J,N,M
115      WRITE(2,115)N,M
      FORMAT(10X,'N = ',I5, 5X,'M = ',I5)
      WRITE(2,114)
114      FORMAT(5X)
      WRITE(2,114)
      WRITE(2,114)
      WRITE(2,116)
116      FORMAT(5X,'      ROW      COL')
      DO 130 I=1,N
        DO 131 J=1,M
          WRITE(2,132)I,J,A(I,J)
132      FORMAT(5X,I4,5X,I4,5X,F8.6,5X,F8.6)
131      CONTINUE
          WRITE(2,114)
          WRITE(2,114)
130      CONTINUE
      RETURN
      END

```

```

C=SUBROUTINE=WRTT
      SUBROUTINE WRTT(NAME,A,N,M)
      COMPLEX A(500,500)
      INTEGER I,J,N,M
      CHARACTER NAME*80
      WRITE(2,125)
      WRITE(2,125)
      WRITE(2,125)
125      FORMAT('*****')
      +*****')
      WRITE(2,114)
      WRITE(2,120)NAME
120      FORMAT(5X,A50)
      WRITE(2,114)

```

```

        WRITE(2,114)
        WRITE(2,115)N,M
115      FORMAT(10X,'NUMBER OF ROWS, N = ',I5,
+5X,'NUMBER OF COLUMN, M = ',I5)
        WRITE(2,114)
114      FORMAT(5X)
        WRITE(2,114)
        WRITE(2,114)
        WRITE(2,116)
116      FORMAT(5X,'      ROW      COL
+      REAL
+      IMAGINARY')
        WRITE(2,117)
117      FORMAT(7X,'*****',4X,'*****',10X,
+ '*****',8X,'*****')
        DO 130 I=1,N
        DO 131 J=1,M
        WRITE(2,132)I,J,A(I,J)
132      FORMAT(5X,I4,5X,I4,5X,E20.9,5X,E20.9)
131      CONTINUE
        WRITE(2,114)
        WRITE(2,114)
130      CONTINUE
        RETURN
        END

```

```

C=SUBROUTINE=ADD
      SUBROUTINE ADD(A,B,C,M,N)

```

```

C
C* * * * *
C
C      MATRIX ADDITION   ( A = B + C )
C
C* * * * *
C
C      COMPLEX A,B,C
C      INTEGER I,J,M,N
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      DIMENSION A(M,N),B(M,N),C(M,N)
C      COMPLEX A(500,500),B(500,500),C(500,500)

```

es:vax

```

        DO 1 I=1,M
        DO 2 J=1,N
        A(I,J)=B(I,J)+C(I,J)
2 CONTINUE
1 CONTINUE
        RETURN
        END

```

```

C=SUBROUTINE=SUBTT
      SUBROUTINE SUBTT(A,B,C,M,N)

```

```

C
C* * * * *
C
C      MATRIX ADDITION   ( A = B - C )
C
C* * * * *

```

```

C
C   COMPLEX A,B,C
C   INTEGER I,J,M,N
C   IMPLICIT REAL*8 (A-H,O-Z)
C
C   DIMENSION A(M,N),B(M,N),C(M,N)
C   COMPLEX A(500,500),B(500,500),C(500,500)

```

es:vax

```

      DO 1 I=1,M
      DO 2 J=1,N
      A(I,J)=B(I,J)-C(I,J)
2    CONTINUE
1    CONTINUE
      RETURN
      END

```

```

C=SUBROUTINE=MULTT
      SUBROUTINE MULTT(A,B,C,L,M,N)

```

M

```

C
C* * * * *
C
C   MATRIX MULTIPLICATION   ( A = B * CT )
C
C* * * * *

```

```

C   IMPLICIT REAL*8 (A-H,O-Z)
C
C   DIMENSION A(L,N),B(L,M),C(N,M)
C   DO 1 J=1,N
C     DO 1 K=1,M
C       IF(C(J,K).EQ.0.DO) GOTO 1
C       DO 2 I=1,L
C         A(I,J)=A(I,J)+B(I,K)*C(J,K)
2     CONTINUE
1   CONTINUE
      RETURN
      END

```

es:vax

```

C=SUBROUTINE=TRANSP   CALLED BY 'CNODEL'
      SUBROUTINE TRANSP(A,N)

```

```

C
C *****
C
C   THIS SUBROUTINE TRANSPOSES AN N * N MATRIX ONTO ITSELF
C
C *****
C

```

```

      COMPLEX A,AH
      DIMENSION A(N,N)
      IF(N.LE.1) RETURN
      N1=N-1
      DO 1 I=1,N1
      I1=I+1
      DO 1 J=I1,N
      AH=A(I,J)
      A(I,J)=A(J,I)

```



```

A(J,I)=AH
1 CONTINUE
RETURN
END

SUBROUTINE MATINVC(A,B,NROW,NR)
COMPLEX A(500,500),B(500,500)
COMPLEX DUM,FACTOR
INTEGER I,K,NROW,I1,J,J1

K = 0
DO 10 I = 1 , NROW
  K = K + 1
  A(I, (NROW + K)) = 1.
10 CONTINUE

DO 20 I = 1 , (NROW - 1)

  B(I, I) = A(I, I)
  DO 30 I1 = (I + 1), NROW

    FACTOR = A(I1, I) / A(I, I)
    DO 40 J1 = 1, NROW

      A(I1, J1) = A(I1, J1) - FACTOR * A(I, J1)
      A(I1, (J1+NROW)) = A(I1, (J1+NROW)) - FACTOR * A(I, (J1+NROW))
40 CONTINUE
30 CONTINUE
20 CONTINUE

DO 50 I = NROW , 2 , -1
  DO 60 I1 = 1 , (I - 1)

    FACTOR = A(I1, I) / A(I, I)

    DO 70 J1 = 1 , NROW

      A(I1, J1) = A(I1, J1) - FACTOR * A(I, J1)
      A(I1, (J1+NROW)) = A(I1, (J1+NROW)) - FACTOR * A(I, (J1+NROW))
70 CONTINUE
60 CONTINUE
50 CONTINUE

DO 80 I = 1 , NROW
  B(I, I) = A(I, I)
80 CONTINUE

DO 90 I = 1 , NROW
  DO 100 J = 1 , NROW
    A(I, J) = A(I, J) / B(I, I)
    A(I, (J + NROW)) = A(I, (J + NROW)) / B(I, I)
100 CONTINUE
90 CONTINUE

C WRITE(*,110)
110 FORMAT(5X,'ROW',5X,'COLUMN',5X,'INVERSE(I,J)')
DO 120 I = 1 , NROW

```

```

DO 130 J = 1 , NROW
  B(I,J)=A(I,(J+NROW))
  WRITE(*,140)I, J, A(I, (J + NROW))
C140  FORMAT(5X,I5,5X,I5,5X,F10.6,5X,F10.6)
  C  WRITE(*,141)I, J, B(I, J)
C141  FORMAT(20X,I5,5X,I5,5X,F10.6,5X,F10.6)
130  CONTINUE
120  CONTINUE
  RETURN
  END

```

```

SUBROUTINE MULT(A,B,C,NL,NM,NN)
C  A(NL,NN)=B(NL,NM)*C(NM,NN)
  COMPLEX A(500,500),B(500,500),C(500,500)
  COMPLEX DUM
  INTEGER I,J,K,L,M,N,NL,NM,NN
  DO 20 I=1,NN
    DO 16 N=1,NL
      DUM=(0.,0.)
      DO 17 J=1,NM
        DUM=DUM+C(J,I)*B(N,J)
17      CONTINUE
      A(N,I)=DUM
16    CONTINUE
20  CONTINUE

```

```

  RETURN
  END

```

```

SUBROUTINE RNAS(KAA,MAA,CAA,N)
  REAL KAA(500,500),MAA(500,500),CAA(500,500),DAA(500,500),VAL(5)
  INTEGER NROW,NNROW,NCOL,I,II,K,K1,K2,K3,J,NBEG,NUM
C  OPEN(UNIT=1,FILE='T1.F21',STATUS='OLD')
C  OPEN(UNIT=2,FILE='OUT.DAT',status='new')

  DO 1 I=1,N
    DO 2 J=1,N
      KAA(I,J)=0.
      MAA(I,J)=0.
      CAA(I,J)=0.
      DAA(I,J)=0.
2    CONTINUE
1  CONTINUE

  DO 99 II=1,3
    READ(1,17)NROW,NCOL
C    WRITE(*,17)NROW,NCOL
17  FORMAT(2I8)
    DO 100 I=1,NROW
      READ(1,18)NNROW,NBEG,NUM
C      WRITE(*,18)NNROW,NBEG,NUM
18  FORMAT(3I8)

      IF (NUM .LE. 5) THEN

        READ(1,20)VAL(1),VAL(2),VAL(3),VAL(4),VAL(5)
C        WRITE(*,20)VAL(1),VAL(2),VAL(3),VAL(4),VAL(5)
20      FORMAT(5E16.9)

```

```

      K3=0
      K1=NBEG
      K2=K1+NUM-1
      DO 200 J=K1,K2
        K3=K3+1
        DAA(NNROW,J)=VAL(K3)
        IF(II.EQ.1) KAA(NNROW,J)=DAA(NNROW,J)
        IF(II.EQ.2) MAA(NNROW,J)=DAA(NNROW,J)
        IF(II.EQ.3) CAA(NNROW,J)=DAA(NNROW,J)
        WRITE(2,235) NNROW,J,DAA(NNROW,J)
C      235      FORMAT(2I5,5X,E16.9)
      200      CONTINUE

      ELSE

        K1 = NBEG
        K2=K1+5-1
      210      READ(1,20) VAL(1), VAL(2), VAL(3), VAL(4), VAL(5)
C      WRITE(*,20) VAL(1), VAL(2), VAL(3), VAL(4), VAL(5)
        K3=0
        DO 205 J=K1,K2
          K3=K3+1
          DAA(NNROW,J)=VAL(K3)
          IF(II.EQ.1) KAA(NNROW,J)=DAA(NNROW,J)
          IF(II.EQ.2) MAA(NNROW,J)=DAA(NNROW,J)
          IF(II.EQ.3) CAA(NNROW,J)=DAA(NNROW,J)
C      WRITE(2,235) NNROW,J,DAA(NNROW,J)
      205      CONTINUE
        NUM = NUM - 5
        IF (NUM .GE. 5) THEN
          K1=K1+5
          GO TO 210
        ELSE
          IF (NUM .EQ. 0) GO TO 100
          READ(1,20) VAL(1), VAL(2), VAL(3), VAL(4), VAL(5)
C      WRITE(*,20) VAL(1), VAL(2), VAL(3), VAL(4), VAL(5)
          K1=K2+1
          K2=K1+NUM-1
          K3 = 0
          DO 220 J=K1,K2
            K3 = K3 + 1
            DAA(NNROW,J)=VAL(K3)
            IF(II.EQ.1) KAA(NNROW,J)=DAA(NNROW,J)
            IF(II.EQ.2) MAA(NNROW,J)=DAA(NNROW,J)
            IF(II.EQ.3) CAA(NNROW,J)=DAA(NNROW,J)
C      WRITE(2,235) NNROW,J,DAA(NNROW,J)
      220      CONTINUE
          ENDDIF
        ENDDIF
      100      CONTINUE

C      IF(II.EQ.3) GO TO 171
      READ(1,19) DUM
      READ(1,19) DUM
      19      FORMAT(20A)

      99      CONTINUE

```

RETURN  
END

SUBROUTINE LZIT(N, A, NA, B, NB, X, NX, WANTX, ITER, EIGA,  
\* EIGB)

LZI 10

C THIS SUBROUTINE SOLVES THE GENERALIZED EIGENVALUE PROBLEM  
C  $A X = \text{LAMBDA } B X$   
C WHERE A IS A COMPLEX UPPER HESSENBERG MATRIX OF  
C ORDER N AND B IS A COMPLEX UPPER TRIANGULAR MATRIX OF ORDER N  
C INPUT PARAMETERS  
C N ORDER OF A AND B  
C A AN N X N UPPER HESSENBERG COMPLEX MATRIX  
C NA THE ROW DIMENSION OF THE A MATRIX  
C B AN N X N UPPER TRIANGULAR COMPLEX MATRIX  
C NB THE ROW DIMENSION OF THE B MATRIX  
C X CONTAINS TRANSFORMATIONS TO OBTAIN EIGENVECTORS OF  
C ORIGINAL SYSTEM. IF EIGENVECTORS ARE REQUESTED AND QZHES  
C IS NOT CALLED, X SHOULD BE SET TO THE IDENTITY MATRIX  
C NX THE ROW DIMENSION OF THE X MATRIX  
C WANTX LOGICAL VARIABLE WHICH SHOULD BE SET TO .TRUE.  
C IF EIGENVECTORS ARE WANTED. OTHERWISE IT  
C SHOULD BE SET TO .FALSE.  
C OUTPUT PARAMETERS  
C X THE ITH COLUMN CONTAINS THE ITH EIGENVECTOR  
C IF EIGENVECTORS ARE REQUESTED  
C ITER AN INTEGER ARRAY OF LENGTH N WHOSE ITH ENTRY  
C CONTAINS THE NUMBER OF ITERATIONS NEEDED TO FIND  
C THE ITH EIGENVALUE. FOR ANY I IF ITER(I) = -1 THEN  
C AFTER 30 ITERATIONS THERE HAS NOT BEEN A SUFFICIENT  
C DECREASE IN THE LAST SUBDIAGONAL ELEMENT OF A  
C TO CONTINUE ITERATING.  
C EIGA A COMPLEX ARRAY OF LENGTH N CONTAINING THE DIAGONAL OF A  
C EIGB A COMPLEX ARRAY OF LENGTH N CONTAINING THE DIAGONAL OF B  
C THE ITH EIGENVALUE CAN BE FOUND BY DIVIDING EIGA(I) BY  
C EIGB(I). WATCH OUT FOR EIGB(I) BEING ZERO  
C COMPLEX A(NA,N), B(NB,N), EIGA(N), EIGB(N)  
C COMPLEX S, W, Y, Z, CSQRT  
C COMPLEX X(NX,N)  
C INTEGER ITER(N)  
C COMPLEX ANNM1, ALFM, BETM, D, SL, DEN, NUM, ANM1M1  
C REAL EPSA, EPSB, SS, R, ANORM, BNORM, ANI, BNI, C  
C REAL D0, D1, D2, E0, E1  
C LOGICAL WANTX  
C NN = N  
C COMPUTE THE MACHINE PRECISION TIMES THE NORM OF A AND B  
C ANORM = 0.  
C BNORM = 0.  
C DO 30 I=1,N  
C ANI = 0.  
C IF (I.EQ.1) GO TO 10  
C Y = A(I,I-1)  
C ANI = ANI + ABS(REAL(Y)) + ABS(AIMAG(Y))  
10 BNI = 0.  
C DO 20 J=I,N  
C ANI = ANI + ABS(REAL(A(I,J))) + ABS(AIMAG(A(I,J)))  
C BNI = BNI + ABS(REAL(B(I,J))) + ABS(AIMAG(B(I,J)))  
20 CONTINUE  
C IF (ANI.GT.ANORM) ANORM = ANI  
C IF (BNI.GT.BNORM) BNORM = BNI  
30 CONTINUE

```

      IF (ANORM.EQ.0.) ANORM = 1.0
      IF (BNORM.EQ.0.) BNORM = 1.0
      EPSB = BNORM
      EPSA = ANORM
40  EPSA = EPSA/2.0
      EPSB = EPSB/2.0
      C = ANORM + EPSA
      IF (C.GT.ANORM) GO TO 40
      IF (N.LE.1) GO TO 320
50  ITS = 0
      NM1 = NN - 1
C CHECK FOR NEGLIGIBLE SUBDIAGONAL ELEMENTS
60  D2 = ABS(REAL(A(NN,NN))) + ABS(AIMAG(A(NN,NN)))
      DO 70 LB=2,NN
          L = NN + 2 - LB
          SS = D2
          Y = A(L-1,L-1)
          D2 = ABS(REAL(Y)) + ABS(AIMAG(Y))
          SS = SS + D2
          Y = A(L,L-1)
          R = SS + ABS(REAL(Y)) + ABS(AIMAG(Y))
          IF (R.EQ.SS) GO TO 80
70  CONTINUE
      L = 1
80  IF (L.EQ.NN) GO TO 320
      IF (ITS.LT.30) GO TO 90
      ITER(NN) = -1
      IF (ABS(REAL(A(NN,NM1)))+ABS(AIMAG(A(NN,NM1))).GT.0.8*
        * ABS(REAL(ANNM1))+ABS(AIMAG(ANNM1))) RETURN
90  IF (ITS.EQ.10 .OR. ITS.EQ.20) GO TO 110
C COMPUTE SHIFT AS EIGENVALUE OF LOWER 2 BY 2
      ANNM1 = A(NN,NM1)
      ANNM1M1 = A(NM1,NM1)
      S = A(NN,NN)*B(NM1,NM1) - ANNM1*B(NM1,NN)
      W = ANNM1*B(NN,NN)*(A(NM1,NN)*B(NM1,NM1)-B(NM1,NN)*ANNM1M1)
      Y = (ANNM1M1*B(NN,NN)-S)/2.
      Z = CSQRT(Y*Y+W)
      IF (REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0) GO TO 100
      DO = REAL(Y/Z)
      IF (DO.LT.0.0) Z = -Z
100  DEN = (Y+Z)*B(NM1,NM1)*B(NN,NN)
      IF (REAL(DEN).EQ.0.0 .AND. AIMAG(DEN).EQ.0.0) DEN =
        * CMPLX(EPSA,0.0)
      NUM = (Y+Z)*S - W
      GO TO 120
C AD-HOC SHIFT
110  Y = A(NM1,NN-2)
      NUM = CMPLX(ABS(REAL(ANNM1))+ABS(AIMAG(ANNM1)),ABS(REAL(Y))
        * +ABS(AIMAG(Y)))
      DEN = (1.0,0.0)
C CHECK FOR 2 CONSECUTIVE SMALL SUBDIAGONAL ELEMENTS
120  IF (NN.EQ.L+1) GO TO 140
      D2 = ABS(REAL(A(NM1,NM1))) + ABS(AIMAG(A(NM1,NM1)))
      E1 = ABS(REAL(ANNM1)) + ABS(AIMAG(ANNM1))
      D1 = ABS(REAL(A(NN,NN))) + ABS(AIMAG(A(NN,NN)))
      NL = NN - (L+1)
      DO 130 MB=1,NL
          M = NN - MB
          E0 = E1
          Y = A(M,M-1)

```

```

      E1 = ABS(REAL(Y)) + ABS(AIMAG(Y))
      D0 = D1
      D1 = D2
      Y = A(M-1,M-1)
      D2 = ABS(REAL(Y)) + ABS(AIMAG(Y))
      Y = A(M,M)*DEN - B(M,M)*NUM
      D0 = (D0+D1+D2)*(ABS(REAL(Y))+ABS(AIMAG(Y)))
      E0 = E0*E1*(ABS(REAL(DEN))+ABS(AIMAG(DEN))) + D0
      IF (E0.EQ.D0) GO TO 150
130 CONTINUE
140 M = L
150 CONTINUE
      ITS = ITS + 1
      W = A(M,M)*DEN - B(M,M)*NUM
      Z = A(M+1,M)*DEN
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
C FIND L AND M AND SET A=LAM AND B=LBM
C
      NP1 = N + 1
      LOR1 = L
      NNORN = NN
      IF (.NOT.WANTX) GO TO 160
      LOR1 = 1
      NNORN = N
160 DO 310 I=M,NM1
      J = I + 1
C FIND ROW TRANSFORMATIONS TO RESTORE A TO
C UPPER HESSENBERG FORM. APPLY TRANSFORMATIONS
C TO A AND B
      IF (I.EQ.M) GO TO 170
      W = A(I,I-1)
      Z = A(J,I-1)
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
      IF (D1.EQ.0.0) GO TO 60
170 IF (D2.GT.D1) GO TO 190
C MUST INTERCHANGE ROWS
      DO 180 K=I,NNORN
      Y = A(I,K)
      A(I,K) = A(J,K)
      A(J,K) = Y
      Y = B(I,K)
      B(I,K) = B(J,K)
      B(J,K) = Y
180 CONTINUE
      IF (I.GT.M) A(I,I-1) = A(J,I-1)
      IF (D2.EQ.0.0) GO TO 220
C THE SCALING OF W AND Z IS DESIGNED TO AVOID A DIVISION BY ZERO
C WHEN THE DENOMINATOR IS SMALL
      Y = CMPLX(REAL(W)/D1,AIMAG(W)/D1)/CMPLX(REAL(Z)/D1,AIMAG(Z)/
      * D1)
      GO TO 200
190 Y = CMPLX(REAL(Z)/D2,AIMAG(Z)/D2)/CMPLX(REAL(W)/D2,AIMAG(W)/
      * D2)
200 DO 210 K=I,NNORN
      A(J,K) = A(J,K) - Y*A(I,K)
      B(J,K) = B(J,K) - Y*B(I,K)
210 CONTINUE
220 IF (I.GT.M) A(J,I-1) = (0.0,0.0)
C PERFORM TRANSFORMATIONS FROM RIGHT TO RESTORE B TO

```

```

C   TRIANGULAR FORM
C   APPLY TRANSFORMATIONS TO A
      Z = B(J,I)
      W = B(J,J)
      D2 = ABS(REAL(W)) + ABS(AIMAG(W))
      D1 = ABS(REAL(Z)) + ABS(AIMAG(Z))
      IF (D1.EQ.0.0) GO TO 60
      IF (D2.GT.D1) GO TO 270
C MUST INTERCHANGE COLUMNS
      DO 230 K=LOR1,J
        Y = A(K,J)
        A(K,J) = A(K,I)
        A(K,I) = Y
        Y = B(K,J)
        B(K,J) = B(K,I)
        B(K,I) = Y
230   CONTINUE
      IF (I.EQ.NM1) GO TO 240
      Y = A(J+1,J)
      A(J+1,J) = A(J+1,I)
      A(J+1,I) = Y
240   IF (.NOT.WANTX) GO TO 260
      DO 250 K=1,N
        Y = X(K,J)
        X(K,J) = X(K,I)
        X(K,I) = Y
250   CONTINUE
260   IF (D2.EQ.0.0) GO TO 310
      Z = CMPLX(REAL(W)/D1,AIMAG(W)/D1)/CMPLX(REAL(Z)/D1,AIMAG(Z)/
      * D1)
      GO TO 280
270   Z = CMPLX(REAL(Z)/D2,AIMAG(Z)/D2)/CMPLX(REAL(W)/D2,AIMAG(W)/
      * D2)
280   DO 290 K=LOR1,J
        A(K,I) = A(K,I) - Z*A(K,J)
        B(K,I) = B(K,I) - Z*B(K,J)
290   CONTINUE
      B(J,I) = (0.0,0.0)
      IF (I.LT.NM1) A(I+2,I) = A(I+2,I) - Z*A(I+2,J)
      IF (.NOT.WANTX) GO TO 310
      DO 300 K=1,N
        X(K,I) = X(K,I) - Z*X(K,J)
300   CONTINUE
310   CONTINUE
      GO TO 60
320   CONTINUE
      EIGA(NN) = A(NN,NN)
      EIGB(NN) = B(NN,NN)
      IF (NN.EQ.1) GO TO 330
      ITER(NN) = ITS
      NN = NM1
      IF (NN.GT.1) GO TO 50
      ITER(1) = 0
      GO TO 320
C FIND EIGENVECTORS USING B FOR INTERMEDIATE STORAGE
330   IF (.NOT.WANTX) RETURN
      M = N
340   CONTINUE
      ALFM = A(M,M)
      BETM = B(M,M)

```

```

      B(M,M) = (1.0,0.0)
      L = M - 1
      IF (L.EQ.0) GO TO 370
350  CONTINUE
      L1 = L + 1
      SL = (0.0,0.0)
      DO 360 J=L1,M
        SL = SL + (BETM*A(L,J)-ALFM*B(L,J))*B(J,M)
360  CONTINUE
      Y = BETM*A(L,L) - ALFM*B(L,L)
      IF (REAL(Y).EQ.0.0 .AND. AIMAG(Y).EQ.0.0) Y =
      * CMPLX((EPSA+EPSB)/2.0,0.0)
      B(L,M) = -SL/Y
      L = L - 1
370  IF (L.GT.0) GO TO 350
      M = M - 1
      IF (M.GT.0) GO TO 340
C TRANSFORM TO ORIGINAL COORDINATE SYSTEM
      M = N
380  CONTINUE
      DO 400 I=1,N
        S = (0.0,0.0)
        DO 390 J=1,M
          S = S + X(I,J)*B(J,M)
390  CONTINUE
        X(I,M) = S
400  CONTINUE
      M = M - 1
      IF (M.GT.0) GO TO 380
C NORMALIZE SO THAT LARGEST COMPONENT = 1.
      M = N
410  CONTINUE
      SS = 0.
      DO 420 I=1,N
        R = ABS(REAL(X(I,M))) + ABS(AIMAG(X(I,M)))
        IF (R.LT.SS) GO TO 420
        SS = R
        D = X(I,M)
420  CONTINUE
      IF (SS.EQ.0.0) GO TO 440
      DO 430 I=1,N
        X(I,M) = X(I,M)/D
430  CONTINUE
440  M = M - 1
      IF (M.GT.0) GO TO 410
      RETURN
      END

```

```

      SUBROUTINE LZHES(N, A, NA, B, NB, X, NX, WANTX)
C THIS SUBROUTINE REDUCES THE COMPLEX MATRIX A TO UPPER
C HESSENBERG FORM AND REDUCES THE COMPLEX MATRIX B TO
C TRIANGULAR FORM
C INPUT PARAMETERS..
C N THE ORDER OF THE A AND B MATRICES
C A A COMPLEX MATRIX
C NA THE ROW DIMENSION OF THE A MATRIX
C B A COMPLEX MATRIX
C NB THE ROW DIMENSION OF THE B MATRIX
C NX THE ROW DIMENSION OF THE X MATRIX

```

LZH 10



```

C WANTX A LOGICAL VARIABLE WHICH IS SET TO .TRUE. IF
C   THE EIGENVECTORS ARE WANTED. OTHERWISE IT SHOULD
C   BE SET TO .FALSE.
C OUTPUT PARAMETERS..
C A ON OUTPUT A IS AN UPPER HESSENBERG MATRIX, THE
C   ORIGINAL MATRIX HAS BEEN DESTROYED
C B AN UPPER TRIANGULAR MATRIX, THE ORIGINAL MATRIX
C   HAS BEEN DESTROYED
C X CONTAINS THE TRANSFORMATIONS NEEDED TO COMPUTE
C   THE EIGENVECTORS OF THE ORIGINAL SYSTEM
C   COMPLEX Y, W, Z, A(NA,N), B(NB,N), X(NX,N)
C   REAL C, D
C   LOGICAL WANTX
C   NM1 = N - 1
C REDUCE B TO TRIANGULAR FORM USING ELEMENTARY
C TRANSFORMATIONS
  DO 80 I=1,NM1
    D = 0.00
    IP1 = I + 1
    DO 10 K=IP1,N
      Y = B(K,I)
      C = ABS(REAL(Y)) + ABS(AIMAG(Y))
      IF (C.LE.D) GO TO 10
      D = C
      II = K
10   CONTINUE
      IF (D.EQ.0.0) GO TO 80
      Y = B(I,I)
      IF (D.LE.ABS(REAL(Y))+ABS(AIMAG(Y))) GO TO 40
C MUST INTERCHANGE
      DO 20 J=1,N
        Y = A(I,J)
        A(I,J) = A(II,J)
        A(II,J) = Y
20   CONTINUE
      DO 30 J=I,N
        Y = B(I,J)
        B(I,J) = B(II,J)
        B(II,J) = Y
30   CONTINUE
40   DO 70 J=IP1,N
        Y = B(J,I)/B(I,I)
        IF (REAL(Y).EQ.0.0 .AND. AIMAG(Y).EQ.0.0) GO TO 70
        DO 50 K=1,N
          A(J,K) = A(J,K) - Y*A(I,K)
50   CONTINUE
          DO 60 K=IP1,N
            B(J,K) = B(J,K) - Y*B(I,K)
60   CONTINUE
70   CONTINUE
      B(IP1,I) = (0.0,0.0)
80 CONTINUE
C INITIALIZE X
  IF (.NOT.WANTX) GO TO 110
  DO 100 I=1,N
    DO 90 J=1,N
      X(I,J) = (0.0,0.0)
90   CONTINUE
      X(I,I) = (1.0,0.00)
100 CONTINUE

```

```

C REDUCE A TO UPPER HESSENBERG FORM
110 NM2 = N - 2
IF (NM2.LT.1) GO TO 270
DO 260 J=1,NM2
  JM2 = NM1 - J
  JP1 = J + 1
  DO 250 II=1,JM2
    I = N + 1 - II
    IM1 = I - 1
    IMJ = I - J
    W = A(I,J)
    Z = A(IM1,J)
    IF (ABS(REAL(W))+ABS(AIMAG(W)).LE.ABS(REAL(Z))
      * +ABS(AIMAG(Z))) GO TO 140
C MUST INTERCHANGE ROWS
    DO 120 K=J,N
      Y = A(I,K)
      A(I,K) = A(IM1,K)
      A(IM1,K) = Y
120    CONTINUE
    DO 130 K=IM1,N
      Y = B(I,K)
      B(I,K) = B(IM1,K)
      B(IM1,K) = Y
130    CONTINUE
140    Z = A(I,J)
    IF (REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0) GO TO 170
    Y = Z/A(IM1,J)
    DO 150 K=JP1,N
      A(I,K) = A(I,K) - Y*A(IM1,K)
150    CONTINUE
    DO 160 K=IM1,N
      B(I,K) = B(I,K) - Y*B(IM1,K)
160    CONTINUE
C TRANSFORMATION FROM THE RIGHT
170    W = B(I,IM1)
    Z = B(I,I)
    IF (ABS(REAL(W))+ABS(AIMAG(W)).LE.ABS(REAL(Z))
      * +ABS(AIMAG(Z))) GO TO 210
C MUST INTERCHANGE COLUMNS
    DO 180 K=1,I
      Y = B(K,I)
      B(K,I) = B(K,IM1)
      B(K,IM1) = Y
180    CONTINUE
    DO 190 K=1,N
      Y = A(K,I)
      A(K,I) = A(K,IM1)
      A(K,IM1) = Y
190    CONTINUE
    IF (.NOT.WANTX) GO TO 210
    DO 200 K=IMJ,N
      Y = X(K,I)
      X(K,I) = X(K,IM1)
      X(K,IM1) = Y
200    CONTINUE
210    Z = B(I,IM1)
    IF (REAL(Z).EQ.0.0 .AND. AIMAG(Z).EQ.0.0) GO TO 250
    Y = Z/B(I,I)
    DO 220 K=1,IM1

```

```

      B(K,IM1) = B(K,IM1) - Y*B(K,I)
220  CONTINUE
      B(I,IM1) = (0.0,0.0)
      DO 230 K=1,N
        A(K,IM1) = A(K,IM1) - Y*A(K,I)
230  CONTINUE
      IF (.NOT.WANTX) GO TO 250
      DO 240 K=IMJ,N
        X(K,IM1) = X(K,IM1) - Y*X(K,I)
240  CONTINUE
250  CONTINUE
      A(JP1+1,J) = (0.0,0.0)
260 CONTINUE
270 RETURN
      END

```

C		HES	10
C	-----	HES	20
C		HES	30
C	SUBROUTINE CQZHES(NM,N,AR,AI,BR,BI,MATZ,ZR,ZI)	HES	40

```

C      INTEGER I,J,K,L,N,K1,LB,L1,NM,NK1,NM1
C      REAL AR(NM,N),AI(NM,N),BR(NM,N),BI(NM,N),ZR(NM,N),ZI(NM,N)
      REAL AR(500,500),AI(500,500),BR(500,500),BI(500,500)
      REAL ZR(500,500),ZI(500,500)
      REAL R,S,T,TI,U1,U2,XI,XR,YI,YR,RHO,U1I
      REAL SQRT,CABS,ABS
      LOGICAL MATZ
      COMPLEX CMPLX

```

C THIS SUBROUTINE IS A COMPLEX ANALOGUE OF THE FIRST STEP OF THE  
 C QZ ALGORITHM FOR SOLVING GENERALIZED MATRIX EIGENVALUE PROBLEMS,  
 C SIAM J. NUMER. ANAL. 10, 241-256(1973) BY MOLER AND STEWART.

C THIS SUBROUTINE ACCEPTS A PAIR OF COMPLEX GENERAL MATRICES AND  
 C REDUCES ONE OF THEM TO UPPER HESSENBERG FORM WITH REAL (AND NON-  
 C NEGATIVE) SUBDIAGONAL ELEMENTS AND THE OTHER TO UPPER TRIANGULAR  
 C FORM USING UNITARY TRANSFORMATIONS. IT IS USUALLY FOLLOWED BY  
 C CQZVAL AND POSSIBLY CQZVEC.

C ON INPUT-

C NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL  
 C ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM  
 C DIMENSION STATEMENT,

C N IS THE ORDER OF THE MATRICES,

C A=(AR,AI) CONTAINS A COMPLEX GENERAL MATRIX,

C B=(BR,BI) CONTAINS A COMPLEX GENERAL MATRIX,

C MATZ SHOULD BE SET TO .TRUE. IF THE RIGHT HAND TRANSFORMATIONS  
 C ARE TO BE ACCUMULATED FOR LATER USE IN COMPUTING  
 C EIGENVECTORS, AND TO .FALSE. OTHERWISE.

C ON OUTPUT-

```

C      A HAS BEEN REDUCED TO UPPER HESSENBERG FORM.  THE ELEMENTS
C      BELOW THE FIRST SUBDIAGONAL HAVE BEEN SET TO ZERO, AND THE
C      SUBDIAGONAL ELEMENTS HAVE BEEN MADE REAL (AND NON-NEGATIVE),
C
C      B HAS BEEN REDUCED TO UPPER TRIANGULAR FORM.  THE ELEMENTS
C      BELOW THE MAIN DIAGONAL HAVE BEEN SET TO ZERO,
C
C      Z=(ZR,ZI) CONTAINS THE PRODUCT OF THE RIGHT HAND
C      TRANSFORMATIONS IF MATZ HAS BEEN SET TO .TRUE.
C      OTHERWISE, Z IS NOT REFERENCED.
C
C      QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,
C      APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY
C
C      -----
C
C      ***** INITIALIZE Z *****
C      IF (.NOT. MATZ) GO TO 10
C
C      DO 3 I = 1, N
C
C          DO 2 J = 1, N
C              ZR(I,J) = 0.0
C              ZI(I,J) = 0.0
C      2  CONTINUE
C
C          ZR(I,I) = 1.0
C      3  CONTINUE
C      ***** REDUCE B TO UPPER TRIANGULAR FORM WITH
C      TEMPORARILY REAL DIAGONAL ELEMENTS *****
C      10 IF (N .LE. 1) GO TO 170
C          NM1 = N - 1
C
C          DO 100 L = 1, NM1
C              L1 = L + 1
C              S = 0.0
C
C              DO 20 I = L, N
C                  S = S + ABS(BR(I,L)) + ABS(BI(I,L))
C      20  CONTINUE
C
C              IF (S .EQ. 0.0) GO TO 100
C              RHO = 0.0
C
C              DO 25 I = L, N
C                  BR(I,L) = BR(I,L) / S
C                  BI(I,L) = BI(I,L) / S
C                  RHO = RHO + BR(I,L)**2 + BI(I,L)**2
C      25  CONTINUE
C
C              R = SQRT(RHO)
C              XR = CABS(CMPLX(BR(L,L),BI(L,L)))
C              IF (XR .EQ. 0.0) GO TO 27
C              RHO = RHO + XR * R
C              U1 = -BR(L,L) / XR
C              U1I = -BI(L,L) / XR
C              YR = R / XR + 1.0
C              BR(L,L) = YR * BR(L,L)
C              BI(L,L) = YR * BI(L,L)
C              GO TO 28

```

```

C
27  BR(L,L) = R
    U1 = -1.0
    U1I = 0.0
C
28  DO 50 J = L1, N
    T = 0.0
    TI = 0.0
C
    DO 30 I = L, N
        T = T + BR(I,L) * BR(I,J) + BI(I,L) * BI(I,J)
        TI = TI + BR(I,L) * BI(I,J) - BI(I,L) * BR(I,J)
30  CONTINUE
C
    T = T / RHO
    TI = TI / RHO
C
    DO 40 I = L, N
        BR(I,J) = BR(I,J) - T * BR(I,L) + TI * BI(I,L)
        BI(I,J) = BI(I,J) - T * BI(I,L) - TI * BR(I,L)
40  CONTINUE
C
    XI = U1 * BI(L,J) - U1I * BR(L,J)
    BR(L,J) = U1 * BR(L,J) + U1I * BI(L,J)
    BI(L,J) = XI
50  CONTINUE
C
    DO 80 J = 1, N
    T = 0.0
    TI = 0.0
C
    DO 60 I = L, N
        T = T + BR(I,L) * AR(I,J) + BI(I,L) * AI(I,J)
        TI = TI + BR(I,L) * AI(I,J) - BI(I,L) * AR(I,J)
60  CONTINUE
C
    T = T / RHO
    TI = TI / RHO
C
    DO 70 I = L, N
        AR(I,J) = AR(I,J) - T * BR(I,L) + TI * BI(I,L)
        AI(I,J) = AI(I,J) - T * BI(I,L) - TI * BR(I,L)
70  CONTINUE
C
    XI = U1 * AI(L,J) - U1I * AR(L,J)
    AR(L,J) = U1 * AR(L,J) + U1I * AI(L,J)
    AI(L,J) = XI
80  CONTINUE
C
    BR(L,L) = R * S
    BI(L,L) = 0.0
C
    DO 90 I = L1, N
        BR(I,L) = 0.0
        BI(I,L) = 0.0
90  CONTINUE
C
100 CONTINUE
C ***** REDUCE A TO UPPER HESSENBERG FORM WITH REAL SUBDIAGONAL
C          ELEMENTS, WHILE KEEPING B TRIANGULAR *****
C

```

```

DO 160 K = 1, NM1
  K1 = K + 1
C ***** SET BOTTOM ELEMENT IN K-TH COLUMN OF A REAL *****
  IF (AI(N,K) .EQ. 0.0) GO TO 105
  R = CABS(CMPLX(AR(N,K),AI(N,K)))
  U1 = AR(N,K) / R
  U1I = AI(N,K) / R
  AR(N,K) = R
  AI(N,K) = 0.0
C
  DO 103 J = K1, N
    XI = U1 * AI(N,J) - U1I * AR(N,J)
    AR(N,J) = U1 * AR(N,J) + U1I * AI(N,J)
    AI(N,J) = XI
  103 CONTINUE
C
  XI = U1 * BI(N,N) - U1I * BR(N,N)
  BR(N,N) = U1 * BR(N,N) + U1I * BI(N,N)
  BI(N,N) = XI
  105 IF (K .EQ. NM1) GO TO 170
  NK1 = NM1 - K
C ***** FOR L=N-1 STEP -1 UNTIL K+1 DO -- *****
  DO 150 LB = 1, NK1
    L = N - LB
    L1 = L + 1
C ***** ZERO A(L+1,K) *****
    S = ABS(AR(L,K)) + ABS(AI(L,K)) + AR(L1,K)
    IF (S .EQ. 0.0) GO TO 150
    U1 = AR(L,K) / S
    U1I = AI(L,K) / S
    U2 = AR(L1,K) / S
    R = SQRT(U1*U1+U1I*U1I+U2*U2)
    U1 = U1 / R
    U1I = U1I / R
    U2 = U2 / R
    AR(L,K) = R * S
    AI(L,K) = 0.0
    AR(L1,K) = 0.0
C
    DO 110 J = K1, N
      XR = AR(L,J)
      XI = AI(L,J)
      YR = AR(L1,J)
      YI = AI(L1,J)
      AR(L,J) = U1 * XR + U1I * XI + U2 * YR
      AI(L,J) = U1 * XI - U1I * XR + U2 * YI
      AR(L1,J) = U1 * YR - U1I * YI - U2 * XR
      AI(L1,J) = U1 * YI + U1I * YR - U2 * XI
    110 CONTINUE
C
    XR = BR(L,L)
    BR(L,L) = U1 * XR
    BI(L,L) = -U1I * XR
    BR(L1,L) = -U2 * XR
C
    DO 120 J = L1, N
      XR = BR(L,J)
      XI = BI(L,J)
      YR = BR(L1,J)
      YI = BI(L1,J)

```

```

BR(L,J) = U1 * XR + U1I * XI + U2 * YR
BI(L,J) = U1 * XI - U1I * XR + U2 * YI
BR(L1,J) = U1 * YR - U1I * YI - U2 * XR
BI(L1,J) = U1 * YI + U1I * YR - U2 * XI
120 CONTINUE
C ***** ZERO B(L+1,L) *****
S = ABS(BR(L1,L1)) + ABS(BI(L1,L1)) + ABS(BR(L1,L))
IF (S .EQ. 0.0) GO TO 150
U1 = BR(L1,L1) / S
U1I = BI(L1,L1) / S
U2 = BR(L1,L) / S
R = SQRT(U1*U1+U1I*U1I+U2*U2)
U1 = U1 / R
U1I = U1I / R
U2 = U2 / R
BR(L1,L1) = R * S
BI(L1,L1) = 0.0
BR(L1,L) = 0.0
C
DO 130 I = 1, L
XR = BR(I,L1)
XI = BI(I,L1)
YR = BR(I,L)
YI = BI(I,L)
BR(I,L1) = U1 * XR + U1I * XI + U2 * YR
BI(I,L1) = U1 * XI - U1I * XR + U2 * YI
BR(I,L) = U1 * YR - U1I * YI - U2 * XR
BI(I,L) = U1 * YI + U1I * YR - U2 * XI
130 CONTINUE
C
DO 140 I = 1, N
XR = AR(I,L1)
XI = AI(I,L1)
YR = AR(I,L)
YI = AI(I,L)
AR(I,L1) = U1 * XR + U1I * XI + U2 * YR
AI(I,L1) = U1 * XI - U1I * XR + U2 * YI
AR(I,L) = U1 * YR - U1I * YI - U2 * XR
AI(I,L) = U1 * YI + U1I * YR - U2 * XI
140 CONTINUE
C
IF (.NOT. MATZ) GO TO 150
C
DO 145 I = 1, N
XR = ZR(I,L1)
XI = ZI(I,L1)
YR = ZR(I,L)
YI = ZI(I,L)
ZR(I,L1) = U1 * XR + U1I * XI + U2 * YR
ZI(I,L1) = U1 * XI - U1I * XR + U2 * YI
ZR(I,L) = U1 * YR - U1I * YI - U2 * XR
ZI(I,L) = U1 * YI + U1I * YR - U2 * XI
145 CONTINUE
C
150 CONTINUE
C
160 CONTINUE
C
170 RETURN
C ***** LAST CARD OF CQZHS *****

```





```

C      POSITIVE VALUE OF EPS1 MAY RESULT IN FASTER EXECUTION,
C      BUT LESS ACCURATE RESULTS,
C
C      MATZ SHOULD BE SET TO .TRUE. IF THE RIGHT HAND TRANSFORMATIONS
C      ARE TO BE ACCUMULATED FOR LATER USE IN COMPUTING
C      EIGENVECTORS, AND TO .FALSE. OTHERWISE,
C
C      Z=(ZR,ZI) CONTAINS, IF MATZ HAS BEEN SET TO .TRUE., THE
C      TRANSFORMATION MATRIX PRODUCED IN THE REDUCTION
C      BY CQZHE, IF PERFORMED, OR ELSE THE IDENTITY MATRIX.
C      IF MATZ HAS BEEN SET TO .FALSE., Z IS NOT REFERENCED.
C
C      ON OUTPUT-
C
C      A HAS BEEN REDUCED TO UPPER TRIANGULAR FORM. THE ELEMENTS
C      BELOW THE MAIN DIAGONAL HAVE BEEN SET TO ZERO,
C
C      B IS STILL IN UPPER TRIANGULAR FORM, ALTHOUGH ITS ELEMENTS
C      HAVE BEEN ALTERED. IN PARTICULAR, ITS DIAGONAL HAS BEEN SET
C      REAL AND NON-NEGATIVE. THE LOCATION BR(N,1) IS USED TO
C      STORE EPS1 TIMES THE NORM OF B FOR LATER USE BY CQZVEC,
C
C      ALFR AND ALFI CONTAIN THE REAL AND IMAGINARY PARTS OF THE
C      DIAGONAL ELEMENTS OF THE TRIANGULARIZED A MATRIX,
C
C      BETA CONTAINS THE REAL NON-NEGATIVE DIAGONAL ELEMENTS OF THE
C      CORRESPONDING B. THE GENERALIZED EIGENVALUES ARE THEN
C      THE RATIOS ((ALFR+I*ALFI)/BETA),
C
C      Z CONTAINS THE PRODUCT OF THE RIGHT HAND TRANSFORMATIONS
C      (FOR BOTH STEPS) IF MATZ HAS BEEN SET TO .TRUE.,
C
C      IERR IS SET TO
C      ZERO      FOR NORMAL RETURN,
C      J          IF AR(J,J-1) HAS NOT BECOME
C                ZERO AFTER 50 ITERATIONS.
C
C      QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,
C      APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY
C
C      -----
C
C      IERR = 0
C      ***** COMPUTE EPSA,EPSB *****
C      ANORM = 0.0
C      BNORM = 0.0
C
C      DO 30 I = 1, N
C        ANI = 0.0
C        IF (I .NE. 1) ANI = ABS(AR(I,I-1))
C        BNI = 0.0
C
C        DO 20 J = I, N
C          ANI = ANI + ABS(AR(I,J)) + ABS(AI(I,J))
C          BNI = BNI + ABS(BR(I,J)) + ABS(BI(I,J))
20      CONTINUE
C
C        IF (ANI .GT. ANORM) ANORM = ANI
C        IF (BNI .GT. BNORM) BNORM = BNI
30      CONTINUE

```

```

C      IF (ANORM .EQ. 0.0) ANORM = 1.0
      IF (BNORM .EQ. 0.0) BNORM = 1.0
      EP = EPS1
      IF (EP .GT. 0.0) GO TO 50
C      ***** COMPUTE ROUNDOFF LEVEL IF EPS1 IS ZERO *****
      EP = 1.0
40     EP = EP / 2.0
      IF (1.0 + EP .GT. 1.0) GO TO 40
50     EPSA = EP * ANORM
      EPSB = EP * BNORM
C      ***** REDUCE A TO TRIANGULAR FORM, WHILE
C      KEEPING B TRIANGULAR *****
      LOR1 = 1
      ENORN = N
      EN = N
C      ***** BEGIN QZ STEP *****
60     IF (EN .EQ. 0) GO TO 1001
      IF (.NOT. MATZ) ENORN = EN
      ITS = 0
      NA = EN - 1
      ENM2 = NA - 1
C      ***** CHECK FOR CONVERGENCE OR REDUCIBILITY.
C      FOR L=EN STEP -1 UNTIL 1 DO -- *****
70     DO 80 LL = 1, EN
          LM1 = EN - LL
          L = LM1 + 1
          IF (L .EQ. 1) GO TO 95
          IF (ABS(AR(L,LM1)) .LE. EPSA) GO TO 90
80     CONTINUE
C
90     AR(L,LM1) = 0.0
C      ***** SET DIAGONAL ELEMENT AT TOP OF B REAL *****
95     B11 = CABS(CMPLX(BR(L,L),BI(L,L)))
      IF (B11 .EQ. 0.0) GO TO 98
      U1 = BR(L,L) / B11
      U1I = BI(L,L) / B11
C
      DO 97 J = L, ENORN
          XI = U1 * AI(L,J) - U1I * AR(L,J)
          AR(L,J) = U1 * AR(L,J) + U1I * AI(L,J)
          AI(L,J) = XI
          XI = U1 * BI(L,J) - U1I * BR(L,J)
          BR(L,J) = U1 * BR(L,J) + U1I * BI(L,J)
          BI(L,J) = XI
97     CONTINUE
C
      BI(L,L) = 0.0
98     IF (L .NE. EN) GO TO 100
C      ***** 1-BY-1 BLOCK ISOLATED *****
      ALFR(EN) = AR(EN,EN)
      ALFI(EN) = AI(EN,EN)
      BETA(EN) = B11
      EN = NA
      GO TO 60
C      ***** CHECK FOR SMALL TOP OF B *****
100    L1 = L + 1
      IF (B11 .GT. EPSB) GO TO 120
      BR(L,L) = 0.0
      S = ABS(AR(L,L)) + ABS(AI(L,L)) + ABS(AR(L1,L))

```

```

U1 = AR(L,L) / S
U1I = AI(L,L) / S
U2 = AR(L1,L) / S
R = SQRT(U1*U1+U1I*U1I+U2*U2)
U1 = U1 / R
U1I = U1I / R
U2 = U2 / R
AR(L,L) = R * S
AI(L,L) = 0.0
C
DO 110 J = L1, ENORN
  XR = AR(L,J)
  XI = AI(L,J)
  YR = AR(L1,J)
  YI = AI(L1,J)
  AR(L,J) = U1 * XR + U1I * XI + U2 * YR
  AI(L,J) = U1 * XI - U1I * XR + U2 * YI
  AR(L1,J) = U1 * YR - U1I * YI - U2 * XR
  AI(L1,J) = U1 * YI + U1I * YR - U2 * XI
  XR = BR(L,J)
  XI = BI(L,J)
  YR = BR(L1,J)
  YI = BI(L1,J)
  BR(L1,J) = U1 * YR - U1I * YI - U2 * XR
  BR(L,J) = U1 * XR + U1I * XI + U2 * YR
  BI(L,J) = U1 * XI - U1I * XR + U2 * YI
  BI(L1,J) = U1 * YI + U1I * YR - U2 * XI
110 CONTINUE
C
  LM1 = L
  L = L1
  GO TO 90
C ***** ITERATION STRATEGY *****
120 IF (ITS .EQ. 50) GO TO 1000
  IF (ITS .EQ. 10) GO TO 135
C ***** DETERMINE SHIFT *****
  B33 = BR(NA,NA)
  B33I = BI(NA,NA)
  IF (CABS(CMPLX(B33,B33I)) .GE. EPSB) GO TO 122
  B33 = EPSB
  B33I = 0.0
122 B44 = BR(EN,EN)
  B44I = BI(EN,EN)
  IF (CABS(CMPLX(B44,B44I)) .GE. EPSB) GO TO 124
  B44 = EPSB
  B44I = 0.0
124 B3344 = B33 * B44 - B33I * B44I
  B3344I = B33 * B44I + B33I * B44
  A33 = AR(NA,NA) * B44 - AI(NA,NA) * B44I
  A33I = AR(NA,NA) * B44I + AI(NA,NA) * B44
  A34 = AR(NA,EN) * B33 - AI(NA,EN) * B33I
  X - AR(NA,NA) * BR(NA,EN) + AI(NA,NA) * BI(NA,EN)
  A34I = AR(NA,EN) * B33I + AI(NA,EN) * B33
  X - AR(NA,NA) * BI(NA,EN) - AI(NA,NA) * BR(NA,EN)
  A43 = AR(EN,NA) * B44
  A43I = AR(EN,NA) * B44I
  A44 = AR(EN,EN) * B33 - AI(EN,EN) * B33I - AR(EN,NA) * BR(NA,EN)
  A44I = AR(EN,EN) * B33I + AI(EN,EN) * B33 - AR(EN,NA) * BI(NA,EN)
  SH = A44
  SHI = A44I

```

```

XR = A34 * A43 - A34I * A43I
XI = A34 * A43I + A34I * A43
IF (XR .EQ. 0.0 .AND. XI .EQ. 0.0) GO TO 140
YR = (A33 - SH) / 2.0
YI = (A33I - SHI) / 2.0
Z3 = CSQRT(CMPLX(YR**2-YI**2+XR,2.0*YR*YI+XI))
U1 = REAL(Z3)
U1I = AIMAG(Z3)
IF (YR * U1 + YI * U1I .GE. 0.0) GO TO 125
U1 = -U1
U1I = -U1I
125 Z3 = (CMPLX(SH,SHI) - CMPLX(XR,XI) / CMPLX(YR+U1,YI+U1I))
X / CMPLX(B3344,B3344I)
SH = REAL(Z3)
SHI = AIMAG(Z3)
GO TO 140
C ***** AD HOC SHIFT *****
135 SH = AR(EN,NA) + AR(NA,ENM2)
SHI = 0.0
C ***** DETERMINE ZEROth COLUMN OF A *****
140 A1 = AR(L,L) / B11 - SH
A1I = AI(L,L) / B11 - SHI
A2 = AR(L1,L) / B11
ITS = ITS + 1
IF (.NOT. MATZ) LOR1 = L
C ***** MAIN LOOP *****
DO 260 K = L, NA
K1 = K + 1
K2 = K + 2
KM1 = MAX0(K-1,L)
C ***** ZERO A(K+1,K-1) *****
IF (K .EQ. L) GO TO 170
A1 = AR(K,KM1)
A1I = AI(K,KM1)
A2 = AR(K1,KM1)
170 S = ABS(A1) + ABS(A1I) + ABS(A2)
U1 = A1 / S
U1I = A1I / S
U2 = A2 / S
R = SQRT(U1*U1+U1I*U1I+U2*U2)
U1 = U1 / R
U1I = U1I / R
U2 = U2 / R
C
DO 180 J = KM1, ENORN
XR = AR(K,J)
XI = AI(K,J)
YR = AR(K1,J)
YI = AI(K1,J)
AR(K,J) = U1 * XR + U1I * XI + U2 * YR
AI(K,J) = U1 * XI - U1I * XR + U2 * YI
AR(K1,J) = U1 * YR - U1I * YI - U2 * XR
AI(K1,J) = U1 * YI + U1I * YR - U2 * XI
XR = BR(K,J)
XI = BI(K,J)
YR = BR(K1,J)
YI = BI(K1,J)
BR(K,J) = U1 * XR + U1I * XI + U2 * YR
BI(K,J) = U1 * XI - U1I * XR + U2 * YI
BR(K1,J) = U1 * YR - U1I * YI - U2 * XR

```

```

      BI(K1,J) = U1 * YI + U1I * YR - U2 * XI
180  CONTINUE
C
      IF (K .EQ. L) GO TO 240
      AI(K,KM1) = 0.0
      AR(K1,KM1) = 0.0
      AI(K1,KM1) = 0.0
C ***** ZERO B(K+1,K) *****
240  S = ABS(BR(K1,K1)) + ABS(BI(K1,K1)) + ABS(BR(K1,K))
      U1 = BR(K1,K1) / S
      U1I = BI(K1,K1) / S
      U2 = BR(K1,K) / S
      R = SQRT(U1*U1+U1I*U1I+U2*U2)
      U1 = U1 / R
      U1I = U1I / R
      U2 = U2 / R
      IF (K .EQ. NA) GO TO 245
      XR = AR(K2,K1)
      AR(K2,K1) = U1 * XR
      AI(K2,K1) = -U1I * XR
      AR(K2,K) = -U2 * XR
C
245  DO 250 I = LOR1, K1
      XR = AR(I,K1)
      XI = AI(I,K1)
      YR = AR(I,K)
      YI = AI(I,K)
      AR(I,K1) = U1 * XR + U1I * XI + U2 * YR
      AI(I,K1) = U1 * XI - U1I * XR + U2 * YI
      AR(I,K) = U1 * YR - U1I * YI - U2 * XR
      AI(I,K) = U1 * YI + U1I * YR - U2 * XI
      XR = BR(I,K1)
      XI = BI(I,K1)
      YR = BR(I,K)
      YI = BI(I,K)
      BR(I,K1) = U1 * XR + U1I * XI + U2 * YR
      BI(I,K1) = U1 * XI - U1I * XR + U2 * YI
      BR(I,K) = U1 * YR - U1I * YI - U2 * XR
      BI(I,K) = U1 * YI + U1I * YR - U2 * XI
250  CONTINUE
C
      BI(K1,K1) = 0.0
      BR(K1,K) = 0.0
      BI(K1,K) = 0.0
      IF (.NOT. MATZ) GO TO 260
C
      DO 255 I = 1, N
      XR = ZR(I,K1)
      XI = ZI(I,K1)
      YR = ZR(I,K)
      YI = ZI(I,K)
      ZR(I,K1) = U1 * XR + U1I * XI + U2 * YR
      ZI(I,K1) = U1 * XI - U1I * XR + U2 * YI
      ZR(I,K) = U1 * YR - U1I * YI - U2 * XR
      ZI(I,K) = U1 * YI + U1I * YR - U2 * XI
255  CONTINUE
C
260  CONTINUE
C ***** SET LAST A SUBDIAGONAL REAL AND END QZ STEP *****
      IF (AI(EN,NA) .EQ. 0.0) GO TO 70

```



```

C      A=(AR,AI) CONTAINS A COMPLEX UPPER TRIANGULAR MATRIX,
C
C      B=(BR,BI) CONTAINS A COMPLEX UPPER TRIANGULAR MATRIX WITH REAL
C      DIAGONAL ELEMENTS. IN ADDITION, LOCATION BR(N,1) CONTAINS
C      THE TOLERANCE QUANTITY (EPSB) COMPUTED AND SAVED IN CQZVAL,
C
C      ALFR, ALFI, AND BETA ARE VECTORS WITH COMPONENTS WHOSE
C      RATIOS ((ALFR+I*ALFI)/BETA) ARE THE GENERALIZED
C      EIGENVALUES. THEY ARE USUALLY OBTAINED FROM CQZVAL,
C
C      Z=(ZR,ZI) CONTAINS THE TRANSFORMATION MATRIX PRODUCED IN THE
C      REDUCTIONS BY CQZHEs AND CQZVAL, IF PERFORMED.
C      IF THE EIGENVECTORS OF THE TRIANGULAR PROBLEM ARE
C      DESIRED, Z MUST CONTAIN THE IDENTITY MATRIX.
C
C      ON OUTPUT-
C
C      A IS UNALTERED,
C
C      B HAS BEEN DESTROYED,
C
C      ALFR, ALFI, AND BETA ARE UNALTERED,
C
C      Z CONTAINS THE EIGENVECTORS. EACH EIGENVECTOR IS NORMALIZED
C      SO THAT THE MODULUS OF ITS LARGEST COMPONENT IS 1.0 .
C
C      QUESTIONS AND COMMENTS SHOULD BE DIRECTED TO B. S. GARBOW,
C      APPLIED MATHEMATICS DIVISION, ARGONNE NATIONAL LABORATORY
C
C      -----
C
C      IF (N .LE. 1) GO TO 1001
C      EPSB = BR(N,1)
C      ***** FOR EN=N STEP -1 UNTIL 2 DO -- *****
C      DO 800 NN = 2, N
C        EN = N + 2 - NN
C        NA = EN - 1
C        ALMR = ALFR(EN)
C        ALMI = ALFI(EN)
C        BETM = BETA(EN)
C      ***** FOR I=EN-1 STEP -1 UNTIL 1 DO -- *****
C      DO 700 II = 1, NA
C        I = EN - II
C        R = 0.0
C        RI = 0.0
C        M = I + 1
C
C      DO 610 J = M, EN
C        T = BETM * AR(I,J) - ALMR * BR(I,J) + ALMI * BI(I,J)
C        TI = BETM * AI(I,J) - ALMR * BI(I,J) - ALMI * BR(I,J)
C        IF (J .EQ. EN) GO TO 605
C        XI = T * BI(J,EN) + TI * BR(J,EN)
C        T = T * BR(J,EN) - TI * BI(J,EN)
C        TI = XI
C      605      R = R + T
C              RI = RI + TI
C      610      CONTINUE
C
C      T = ALMR * BETA(I) - BETM * ALFR(I)
C      TI = ALMI * BETA(I) - BETM * ALFI(I)

```

```

      IF (T .EQ. 0.0 .AND. TI .EQ. 0.0) T = EPSB
      Z3 = CMPLX(R,RI) / CMPLX(T,TI)
      BR(I,EN) = REAL(Z3)
      BI(I,EN) = AIMAG(Z3)
700    CONTINUE
C
800    CONTINUE
C      ***** END BACK SUBSTITUTION.
C      TRANSFORM TO ORIGINAL COORDINATE SYSTEM.
C      FOR J=N STEP -1 UNTIL 2 DO -- *****
DO 880 JJ = 2, N
      J = N + 2 - JJ
      M = J - 1
C
      DO 880 I = 1, N
C
          DO 860 K = 1, M
              ZR(I,J) = ZR(I,J) + ZR(I,K) * BR(K,J) - ZI(I,K) * BI(K,J)
              ZI(I,J) = ZI(I,J) + ZR(I,K) * BI(K,J) + ZI(I,K) * BR(K,J)
860          CONTINUE
C
880    CONTINUE
C      ***** NORMALIZE SO THAT MODULUS OF LARGEST
C      COMPONENT OF EACH VECTOR IS 1 *****
DO 950 J = 1, N
C
      DO 930 I = 1, N
          IF(I .EQ. 1) T5 = ABS(ZR(I,J))
          R = ABS(ZR(I,J))
          IF (R .GE. T5) THEN
              T5 = R
              I5=I
          ELSE
              ENDIF
930    CONTINUE
C
      DO 940 I = 1, N
          T6 = CMPLX(ZR(I,J),ZI(I,J))/CMPLX(ZR(I5,J),ZI(I5,J))
          ZR(I,J) = REAL(T6)
          ZI(I,J) = AIMAG(T6)
940    CONTINUE
C
950    CONTINUE
C
1001   RETURN
C      ***** LAST CARD OF CQZVEC *****
      END

```

```

C      subroutine cg(nm,n,ar,ai,wr,wi,matz,zr,zi,fv1,fv2,fv3,ierr)
C
C      integer n,nm,ls1,ls2,ierr,matz
C      double precision ar(nm,n),ai(nm,n),wr(n),wi(n),zr(nm,n),zi(nm,n),

```



```

      x      fv1(n),fv2(n),fv3(n)
c
c      this subroutine calls the recommended sequence of
c      subroutines from the eigensystem subroutine package (eispack)
c      to find the eigenvalues and eigenvectors (if desired)
c      of a complex general matrix.
c
c      on input
c
c      nm must be set to the row dimension of the two-dimensional
c      array parameters as declared in the calling program
c      dimension statement.
c
c      n is the order of the matrix a=(ar,ai).
c
c      ar and ai contain the real and imaginary parts,
c      respectively, of the complex general matrix.
c
c      matz is an integer variable set equal to zero if
c      only eigenvalues are desired. otherwise it is set to
c      any non-zero integer for both eigenvalues and eigenvectors.
c
c      on output
c
c      wr and wi contain the real and imaginary parts,
c      respectively, of the eigenvalues.
c
c      zr and zi contain the real and imaginary parts,
c      respectively, of the eigenvectors if matz is not zero.
c
c      ierr is an integer output variable set equal to an error
c      completion code described in the documentation for comqr
c      and comqr2. the normal completion code is zero.
c
c      fv1, fv2, and fv3 are temporary storage arrays.
c
c      questions and comments should be directed to burton s. garbow,
c      mathematics and computer science div, argonne national laboratory
c
c      this version dated august 1983.
c
c      -----
c
c      if (n .le. nm) go to 10
c      ierr = 10 * n
c      go to 50
c
c      10 call cbal(nm,n,ar,ai,is1,is2,fv1)
c      call corth(nm,n,is1,is2,ar,ai,fv2,fv3)
c      if (matz .ne. 0) go to 20
c      ..... find eigenvalues only .....
c      call comqr(nm,n,is1,is2,ar,ai,wr,wi,ierr)
c      go to 50
c      ..... find both eigenvalues and eigenvectors .....
c      20 call comqr2(nm,n,is1,is2,fv2,fv3,ar,ai,wr,wi,zr,zi,ierr)
c      if (ierr .ne. 0) go to 50
c      call cbabk2(nm,n,is1,is2,fv1,n,zr,zi)
c      50 return
c      end

```

```

subroutine cbabk2(nm,n,low,igh,scale,m,zr,zi)
C
integer i,j,k,m,n,li,nm,igh,low
double precision scale(n),zr(nm,m),zi(nm,m)
double precision s
C
C this subroutine is a translation of the algol procedure
C cbabk2, which is a complex version of balbak,
C num. math. 13, 293-304(1969) by parlett and reinsch.
C handbook for auto. comp., vol.11-linear algebra, 315-326(1971).
C
C this subroutine forms the eigenvectors of a complex general
C matrix by back transforming those of the corresponding
C balanced matrix determined by cbal.
C
on input
C
C nm must be set to the row dimension of two-dimensional
C array parameters as declared in the calling program
C dimension statement.
C
C n is the order of the matrix.
C
C low and igh are integers determined by cbal.
C
C scale contains information determining the permutations
C and scaling factors used by cbal.
C
C m is the number of eigenvectors to be back transformed.
C
C zr and zi contain the real and imaginary parts,
C respectively, of the eigenvectors to be
C back transformed in their first m columns.
C
on output
C
C zr and zi contain the real and imaginary parts,
C respectively, of the transformed eigenvectors
C in their first m columns.
C
C questions and comments should be directed to burton s. garbow,
C mathematics and computer science div, argonne national laboratory
C
C this version dated august 1983.
C
C -----
C
if (m .eq. 0) go to 200
if (igh .eq. low) go to 120
C
do 110 i = low, igh
s = scale(i)
C ..... left hand eigenvectors are back transformed
C if the foregoing statement is replaced by
C s=1.0d0/scale(i). .....
do 100 j = 1, m
zr(i,j) = zr(i,j) * s
zi(i,j) = zi(i,j) * s
100 continue

```

```

C
110 continue
C ..... for i=low-1 step -1 until 1,
C           igh+1 step 1 until n do -- .....
120 do 140 ii = 1, n
    i = ii
    if (i .ge. low .and. i .le. igh) go to 140
    if (i .lt. low) i = low - ii
    k = scale(i)
    if (k .eq. i) go to 140
C
    do 130 j = 1, m
        s = zr(i,j)
        zr(i,j) = zr(k,j)
        zr(k,j) = s
        s = zi(i,j)
        zi(i,j) = zi(k,j)
        zi(k,j) = s
130 continue
C
140 continue
C
200 return
end

subroutine comqr2(nm,n,low,igh,ortr,orti,hr,hi,wr,wi,zr,zi,ierr)
C MESHED overflow control WITH vectors of isolated roots (10/19/89 BSG)
C MESHED overflow control WITH triangular multiply (10/30/89 BSG)
C
integer i,j,k,l,m,n,en,ii,jj,ll,nm,nn,igh,lp1,
x      itn,its,low,lp1,enml,iend,ierr
double precision hr(nm,n),hi(nm,n),wr(n),wi(n),zr(nm,n),zi(nm,n),
x      ortr(igh),orti(igh)
double precision si,sr,ti,tr,xi,xr,yi,yr,zzi,zzr,norm,tst1,tst2,
x      pythag
C
C this subroutine is a translation of a unitary analogue of the
C algol procedure comlr2, num. math. 16, 181-204(1970) by peters
C and wilkinson.
C handbook for auto. comp., vol.11-linear algebra, 372-395(1971).
C the unitary analogue substitutes the qr algorithm of francis
C (comp. jour. 4, 332-345(1962)) for the lr algorithm.
C
C this subroutine finds the eigenvalues and eigenvectors
C of a complex upper hessenberg matrix by the qr
C method. the eigenvectors of a complex general matrix
C can also be found if corth has been used to reduce
C this general matrix to hessenberg form.
C
C on input
C
C nm must be set to the row dimension of two-dimensional
C array parameters as declared in the calling program
C dimension statement.
C
C n is the order of the matrix.
C
C low and igh are integers determined by the balancing
C subroutine cbal. if cbal has not been used,

```

```

C      set low=1, igh=n.
C
C      ortr and orti contain information about the unitary trans-
C      formations used in the reduction by corth, if performed.
C      only elements low through igh are used. if the eigenvectors
C      of the hessenberg matrix are desired, set ortr(j) and
C      orti(j) to 0.0d0 for these elements.
C
C      hr and hi contain the real and imaginary parts,
C      respectively, of the complex upper hessenberg matrix.
C      their lower triangles below the subdiagonal contain further
C      information about the transformations which were used in the
C      reduction by corth, if performed. if the eigenvectors of
C      the hessenberg matrix are desired, these elements may be
C      arbitrary.
C
C      on output
C
C      ortr, orti, and the upper hessenberg portions of hr and hi
C      have been destroyed.
C
C      wr and wi contain the real and imaginary parts,
C      respectively, of the eigenvalues. if an error
C      exit is made, the eigenvalues should be correct
C      for indices ierr+1,...,n.
C
C      zr and zi contain the real and imaginary parts,
C      respectively, of the eigenvectors. the eigenvectors
C      are unnormalized. if an error exit is made, none of
C      the eigenvectors has been found.
C
C      ierr is set to
C      zero      for normal return,
C      j         if the limit of 30*n iterations is exhausted
C               while the j-th eigenvalue is being sought.
C
C      calls cdiv for complex division.
C      calls csroot for complex square root.
C      calls pythag for  $\text{dsqrt}(a^2 + b^2)$  .
C
C      questions and comments should be directed to burton s. garbow,
C      mathematics and computer science div, argonne national laboratory
C
C      this version dated october 1989.
C
C      -----
C
C      ierr = 0
C      ..... initialize eigenvector matrix .....
C      do 101 j = 1, n
C
C          do 100 i = 1, n
C              zr(i,j) = 0.0d0
C              zi(i,j) = 0.0d0
C          100 continue
C              zr(j,j) = 1.0d0
C      101 continue
C      ..... form the matrix of accumulated transformations
C      from the information left by corth .....
C      iend = igh - low - 1

```

```

        if (iend) 180, 150, 105
c      ..... for i=igh-1 step -1 until low+1 do -- .....
105 do 140 ii = 1, iend
        i = igh - ii
        if (ortr(i) .eq. 0.0d0 .and. orti(i) .eq. 0.0d0) go to 140
        if (hr(i,i-1) .eq. 0.0d0 .and. hi(i,i-1) .eq. 0.0d0) go to 140
c      ..... norm below is negative of h formed in corth .....
        norm = hr(i,i-1) * ortr(i) + hi(i,i-1) * orti(i)
        ipl = i + 1
c
        do 110 k = ipl, igh
            ortr(k) = hr(k,i-1)
            orti(k) = hi(k,i-1)
110 continue
c
        do 130 j = 1, igh
            sr = 0.0d0
            si = 0.0d0
c
            do 115 k = i, igh
                sr = sr + ortr(k) * zr(k,j) + orti(k) * zi(k,j)
                si = si + ortr(k) * zi(k,j) - orti(k) * zr(k,j)
115 continue
c
            sr = sr / norm
            si = si / norm
c
            do 120 k = 1, igh
                zr(k,j) = zr(k,j) + sr * ortr(k) - si * orti(k)
                zi(k,j) = zi(k,j) + sr * orti(k) + si * ortr(k)
120 continue
c
130 continue
c
140 continue
c      ..... create real subdiagonal elements .....
150 l = low + 1
c
        do 170 i = 1, igh
            ll = min0(i+1,igh)
            if (hi(i,i-1) .eq. 0.0d0) go to 170
            norm = pythag(hr(i,i-1),hi(i,i-1))
            yr = hr(i,i-1) / norm
            yi = hi(i,i-1) / norm
            hr(i,i-1) = norm
            hi(i,i-1) = 0.0d0
c
            do 155 j = i, n
                si = yr * hi(i,j) - yi * hr(i,j)
                hr(i,j) = yr * hr(i,j) + yi * hi(i,j)
                hi(i,j) = si
155 continue
c
            do 160 j = 1, ll
                si = yr * hi(j,i) + yi * hr(j,i)
                hr(j,i) = yr * hr(j,i) - yi * hi(j,i)
                hi(j,i) = si
160 continue
c
        do 165 j = low, igh

```

```

        si = yr * zi(j,i) + yi * zr(j,i)
        zr(j,i) = yr * zr(j,i) - yi * zi(j,i)
        zi(j,i) = si
165    continue
c
170 continue
c ..... store roots isolated by cbal .....
180 do 200 i = 1, n
        if (i .ge. low .and. i .le. igh) go to 200
        wr(i) = hr(i,i)
        wi(i) = hi(i,i)
200 continue
c
        en = igh
        tr = 0.0d0
        ti = 0.0d0
        itn = 30*n
c ..... search for next eigenvalue .....
220 if (en .lt. low) go to 680
        its = 0
        enml = en - 1
c ..... look for single small sub-diagonal element
c ..... for l=en step -1 until low do -- .....
240 do 260 ll = low, en
        l = en + low - ll
        if (l .eq. low) go to 300
        tst1 = dabs(hr(l-1,l-1)) + dabs(hi(l-1,l-1))
        x      + dabs(hr(l,l)) + dabs(hi(l,l))
        tst2 = tst1 + dabs(hr(l,l-1))
        if (tst2 .eq. tst1) go to 300
260 continue
c ..... form shift .....
300 if (l .eq. en) go to 660
        if (itn .eq. 0) go to 1000
        if (its .eq. 10 .or. its .eq. 20) go to 320
        sr = hr(en,en)
        si = hi(en,en)
        xr = hr(enml,en) * hr(en,enml)
        xi = hi(enml,en) * hr(en,enml)
        if (xr .eq. 0.0d0 .and. xi .eq. 0.0d0) go to 340
        yr = (hr(enml,enml) - sr) / 2.0d0
        yi = (hi(enml,enml) - si) / 2.0d0
        call csroot(yr**2-yi**2+xr,2.0d0*yr*yi+xi,zzr,zzi)
        if (yr * zzr + yi * zzi .ge. 0.0d0) go to 310
        zzr = -zzr
        zzi = -zzi
310 call cdiv(xr,xi,yr+zzr,yi+zzi,xr,xi)
        sr = sr - xr
        si = si - xi
        go to 340
c ..... form exceptional shift .....
320 sr = dabs(hr(en,enml)) + dabs(hr(enml,en-2))
        si = 0.0d0
c
340 do 360 i = low, en
        hr(i,i) = hr(i,i) - sr
        hi(i,i) = hi(i,i) - si
360 continue
c
        tr = tr + sr

```

```

ti = ti + si
its = its + 1
itn = itn - 1
c ..... reduce to triangle (rows) .....
lpl = l + 1
c
do 500 i = lpl, en
  sr = hr(i,i-1)
  hr(i,i-1) = 0.0d0
  norm = pythag(pythag(hr(i-1,i-1),hi(i-1,i-1)),sr)
  xr = hr(i-1,i-1) / norm
  wr(i-1) = xr
  xi = hi(i-1,i-1) / norm
  wi(i-1) = xi
  hr(i-1,i-1) = norm
  hi(i-1,i-1) = 0.0d0
  hi(i,i-1) = sr / norm
c
do 490 j = 1, n
  yr = hr(i-1,j)
  yi = hi(i-1,j)
  z zr = hr(i,j)
  z zi = hi(i,j)
  hr(i-1,j) = xr * yr + xi * yi + hi(i,i-1) * z zr
  hi(i-1,j) = xr * yi - xi * yr + hi(i,i-1) * z zi
  hr(i,j) = xr * z zr - xi * z zi - hi(i,i-1) * yr
  hi(i,j) = xr * z zi + xi * z zr - hi(i,i-1) * yi
490 continue
c
500 continue
c
si = hi(en,en)
if (si .eq. 0.0d0) go to 540
norm = pythag(hr(en,en),si)
sr = hr(en,en) / norm
si = si / norm
hr(en,en) = norm
hi(en,en) = 0.0d0
if (en .eq. n) go to 540
lpl = en + 1
c
do 520 j = lpl, n
  yr = hr(en,j)
  yi = hi(en,j)
  hr(en,j) = sr * yr + si * yi
  hi(en,j) = sr * yi - si * yr
520 continue
c ..... inverse operation (columns) .....
540 do 600 j = lpl, en
  xr = wr(j-1)
  xi = wi(j-1)
c
do 580 i = 1, j
  yr = hr(i,j-1)
  yi = 0.0d0
  z zr = hr(i,j)
  z zi = hi(i,j)
  if (i .eq. j) go to 560
  yi = hi(i,j-1)
  hi(i,j-1) = xr * yi + xi * yr + hi(j,j-1) * z zi

```

```

560      hr(i,j-1) = xr * yr - xi * yi + hi(j,j-1) * z zr
      hr(i,j) = xr * z zr + xi * z zi - hi(j,j-1) * yr
      hi(i,j) = xr * z zi - xi * z zr - hi(j,j-1) * yi
580      continue
c
      do 590 i = low, igh
      yr = zr(i,j-1)
      yi = zi(i,j-1)
      z zr = zr(i,j)
      z zi = zi(i,j)
      zr(i,j-1) = xr * yr - xi * yi + hi(j,j-1) * z zr
      zi(i,j-1) = xr * yi + xi * yr + hi(j,j-1) * z zi
      zr(i,j) = xr * z zr + xi * z zi - hi(j,j-1) * yr
      zi(i,j) = xr * z zi - xi * z zr - hi(j,j-1) * yi
590      continue
c
600      continue
c
      if (si .eq. 0.0d0) go to 240
c
      do 630 i = 1, en
      yr = hr(i,en)
      yi = hi(i,en)
      hr(i,en) = sr * yr - si * yi
      hi(i,en) = sr * yi + si * yr
630      continue
c
      do 640 i = low, igh
      yr = zr(i,en)
      yi = zi(i,en)
      zr(i,en) = sr * yr - si * yi
      zi(i,en) = sr * yi + si * yr
640      continue
c
      go to 240
c
      ..... a root found .....
660      hr(en,en) = hr(en,en) + tr
      wr(en) = hr(en,en)
      hi(en,en) = hi(en,en) + ti
      wi(en) = hi(en,en)
      en = enml
      go to 220
c
      ..... all roots found. backsubstitute to find
c      vectors of upper triangular form .....
680      norm = 0.0d0
c
      do 720 i = 1, n
c
      do 720 j = i, n
      tr = dabs(hr(i,j)) + dabs(hi(i,j))
      if (tr .gt. norm) norm = tr
720      continue
c
      if (n .eq. 1 .or. norm .eq. 0.0d0) go to 1001
c
      ..... for en=n step -1 until 2 do -- .....
      do 800 nn = 2, n
      en = n + 2 - nn
      xr = wr(en)
      xi = wi(en)
      hr(en,en) = 1.0d0

```



```

        hi(en,en) = 0.0d0
        enml = en - 1
c      ..... for i=en-1 step -1 until 1 do -- .....
        do 780 i1 = 1, enml
            i = en - i1
            zzr = 0.0d0
            zzi = 0.0d0
            ip1 = i + 1
c
            do 740 j = ip1, en
                zzr = zzr + hr(i,j) * hr(j,en) - hi(i,j) * hi(j,en)
                zzi = zzi + hr(i,j) * hi(j,en) + hi(i,j) * hr(j,en)
740          continue
c
            yr = xr - wr(i)
            yi = xi - wi(i)
            if (yr .ne. 0.0d0 .or. yi .ne. 0.0d0) go to 765
            tst1 = norm
            yr = tst1
760          yr = 0.01d0 * yr
            tst2 = norm + yr
            if (tst2 .gt. tst1) go to 760
765          continue
            call cdiv(zzr, zzi, yr, yi, hr(i,en), hi(i,en))
c      ..... overflow control .....
            tr = dabs(hr(i,en)) + dabs(hi(i,en))
            if (tr .eq. 0.0d0) go to 780
            tst1 = tr
            tst2 = tst1 + 1.0d0/tst1
            if (tst2 .gt. tst1) go to 780
            do 770 j = 1, en
                hr(j,en) = hr(j,en)/tr
                hi(j,en) = hi(j,en)/tr
770          continue
c
780          continue
c
800 continue
c      ..... end backsubstitution .....
c      ..... vectors of isolated roots .....
        do 840 i = 1, N
            if (i .ge. low .and. i .le. igh) go to 840
c
            do 820 j = 1, n
                zr(i,j) = hr(i,j)
                zi(i,j) = hi(i,j)
820          continue
c
840 continue
c      ..... multiply by transformation matrix to give
c      ..... vectors of original full matrix.
c      ..... for j=n step -1 until low do -- .....
        do 880 jj = low, N
            j = n + low - jj
            m = min0(j, igh)
c
            do 880 i = low, igh
                zzr = 0.0d0
                zzi = 0.0d0
c

```

```

      do 860 k = low, m
        zzr = zzr + zr(i,k) * hr(k,j) - zi(i,k) * hi(k,j)
        zzi = zzi + zr(i,k) * hi(k,j) + zi(i,k) * hr(k,j)
860      continue
c
      zr(i,j) = zzr
      zi(i,j) = zzi
880 continue
c
      go to 1001
c      ..... set error -- all eigenvalues have not
c      converged after 30*n iterations .....
1000 ierr = en
1001 return
      end
      subroutine comqr(nm,n,low,igh,hr,hi,wr,wi,ierr)
c
      integer i,j,l,n,en,ll,nm,igh,ltl,its,low,lpl,enml,ierr
      double precision hr(nm,n),hi(nm,n),wr(n),wi(n)
      double precision si,sr,ti,tr,xi,xr,yi,yr,zzi,zzr,norm,tst1,tst2,
x      pythag
c
c      this subroutine is a translation of a unitary analogue of the
c      algol procedure comlr, num. math. 12, 369-376(1968) by martin
c      and wilkinson.
c      handbook for auto. comp., vol.ii-linear algebra, 396-403(1971).
c      the unitary analogue substitutes the qr algorithm of francis
c      (comp. jour. 4, 332-345(1962)) for the lr algorithm.
c
c      this subroutine finds the eigenvalues of a complex
c      upper hessenberg matrix by the qr method.
c
c      on input
c
c      nm must be set to the row dimension of two-dimensional
c      array parameters as declared in the calling program
c      dimension statement.
c
c      n is the order of the matrix.
c
c      low and igh are integers determined by the balancing
c      subroutine cbal. if cbal has not been used,
c      set low=1, igh=n.
c
c      hr and hi contain the real and imaginary parts,
c      respectively, of the complex upper hessenberg matrix.
c      their lower triangles below the subdiagonal contain
c      information about the unitary transformations used in
c      the reduction by corth, if performed.
c
c      on output
c
c      the upper hessenberg portions of hr and hi have been
c      destroyed. therefore, they must be saved before
c      calling comqr if subsequent calculation of
c      eigenvectors is to be performed.
c
c      wr and wi contain the real and imaginary parts,
c      respectively, of the eigenvalues. if an error
c      exit is made, the eigenvalues should be correct

```

```

c      for indices ierr+1,...,n.
c
c      ierr is set to
c      zero      for normal return,
c      j         if the limit of 30*n iterations is exhausted
c                while the j-th eigenvalue is being sought.
c
c      calls cdiv for complex division.
c      calls csroot for complex square root.
c      calls pythag for dsqrt(a*a + b*b) .
c
c      questions and comments should be directed to burton s. garbow,
c      mathematics and computer science div, argonne national laboratory .
c
c      this version dated august 1983.
c
c      -----
c
c      ierr = 0
c      if (low .eq. igh) go to 180
c      ..... create real subdiagonal elements .....
c      l = low + 1
c
c      do 170 i = l, igh
c          ll = min0(i+1,igh)
c          if (hi(i,i-1) .eq. 0.0d0) go to 170
c          norm = pythag(hr(i,i-1),hi(i,i-1))
c          yr = hr(i,i-1) / norm
c          yi = hi(i,i-1) / norm
c          hr(i,i-1) = norm
c          hi(i,i-1) = 0.0d0
c
c          do 155 j = l, igh
c              si = yr * hi(i,j) - yi * hr(i,j)
c              hr(i,j) = yr * hr(i,j) + yi * hi(i,j)
c              hi(i,j) = si
c          155 continue
c
c          do 160 j = low, ll
c              si = yr * hi(j,i) + yi * hr(j,i)
c              hr(j,i) = yr * hr(j,i) - yi * hi(j,i)
c              hi(j,i) = si
c          160 continue
c
c          170 continue
c      ..... store roots isolated by cbal .....
c      180 do 200 i = l, n
c          if (i .ge. low .and. i .le. igh) go to 200
c          wr(i) = hr(i,i)
c          wi(i) = hi(i,i)
c      200 continue
c
c      en = igh
c      tr = 0.0d0
c      ti = 0.0d0
c      itn = 30*n
c
c      ..... search for next eigenvalue .....
c      220 if (en .lt. low) go to 1001
c          its = 0
c          enml = en - 1

```

```

c ..... look for single small sub-diagonal element
c for l=en step -1 until low d0 -- .....
240 do 260 ll = low, en
      l = en + low - ll
      if (l .eq. low) go to 300
      tst1 = dabs(hr(l-1,l-1)) + dabs(hi(l-1,l-1))
      x      + dabs(hr(l,l)) + dabs(hi(l,l))
      tst2 = tst1 + dabs(hr(l,l-1))
      if (tst2 .eq. tst1) go to 300
260 continue
c ..... form shift .....
300 if (l .eq. en) go to 660
      if (itn .eq. 0) go to 1000
      if (its .eq. 10 .or. its .eq. 20) go to 320
      sr = hr(en,en)
      si = hi(en,en)
      xr = hr(enml,en) * hr(en,enml)
      xi = hi(enml,en) * hr(en,enml)
      if (xr .eq. 0.0d0 .and. xi .eq. 0.0d0) go to 340
      yr = (hr(enml,enml) - sr) / 2.0d0
      yi = (hi(enml,enml) - si) / 2.0d0
      call csroot(yr**2-yi**2+xr,2.0d0*yr*yi+xi,zzr,zzi)
      if (yr * zzr + yi * zzi .ge. 0.0d0) go to 310
      zzr = -zzr
      zzi = -zzi
310 call cdiv(xr,xi,yr+zzr,yi+zzi,xr,xi)
      sr = sr - xr
      si = si - xi
      go to 340
c ..... form exceptional shift .....
320 sr = dabs(hr(en,enml)) + dabs(hr(enml,en-2))
      si = 0.0d0
c
340 do 360 i = low, en
      hr(i,i) = hr(i,i) - sr
      hi(i,i) = hi(i,i) - si
360 continue
c
      tr = tr + sr
      ti = ti + si
      its = its + 1
      itn = itn - 1
c ..... reduce to triangle (rows) .....
      lpl = l + 1
c
do 500 i = lpl, en
      sr = hr(i,i-1)
      hr(i,i-1) = 0.0d0
      norm = pythag(pythag(hr(i-1,i-1),hi(i-1,i-1)),sr)
      xr = hr(i-1,i-1) / norm
      wr(i-1) = xr
      xi = hi(i-1,i-1) / norm
      wi(i-1) = xi
      hr(i-1,i-1) = norm
      hi(i-1,i-1) = 0.0d0
      hi(i,i-1) = sr / norm
c
do 490 j = i, en
      yr = hr(i-1,j)
      yi = hi(i-1,j)

```

```

        zzr = hr(i,j)
        zzi = hi(i,j)
        hr(i-1,j) = xr * yr + xi * yi + hi(i,i-1) * zzr
        hi(i-1,j) = xr * yi - xi * yr + hi(i,i-1) * zzi
        hr(i,j) = xr * zzr - xi * zzi - hi(i,i-1) * yr
        hi(i,j) = xr * zzi + xi * zzr - hi(i,i-1) * yi
490    continue
c
500 continue
c
    si = hi(en,en)
    if (si .eq. 0.0d0) go to 540
    norm = pythag(hr(en,en),si)
    sr = hr(en,en) / norm
    si = si / norm
    hr(en,en) = norm
    hi(en,en) = 0.0d0
c
    ..... inverse operation (columns) .....
540 do 600 j = 1p1, en
        xr = wr(j-1)
        xi = wi(j-1)
c
        do 580 i = 1, j
            yr = hr(i,j-1)
            yi = 0.0d0
            zzr = hr(i,j)
            zzi = hi(i,j)
            if (i .eq. j) go to 560
            yi = hi(i,j-1)
            hi(i,j-1) = xr * yi + xi * yr + hi(j,j-1) * zzi
560        hr(i,j-1) = xr * yr - xi * yi + hi(j,j-1) * zzr
            hr(i,j) = xr * zzr + xi * zzi - hi(j,j-1) * yr
            hi(i,j) = xr * zzi - xi * zzr - hi(j,j-1) * yi
580    continue
c
600 continue
c
    if (si .eq. 0.0d0) go to 240
c
    do 630 i = 1, en
        yr = hr(i,en)
        yi = hi(i,en)
        hr(i,en) = sr * yr - si * yi
        hi(i,en) = sr * yi + si * yr
630 continue
c
    go to 240
c
    ..... a root found .....
660 wr(en) = hr(en,en) + tr
    wi(en) = hi(en,en) + ti
    en = enml
    go to 220
c
    ..... set error -- all eigenvalues have not
c          converged after 30*n iterations .....
1000 ierr = en
1001 return
    end

```

subroutine cdiv(ar,ai,br,bi,cr,ci)

```

double precision ar,ai,br,bi,cr,ci
C
C complex division, (cr,ci) = (ar,ai)/(br,bi)
C
double precision s,ars,ais,brs,bis
s = dabs(br) + dabs(bi)
ars = ar/s
ais = ai/s
brs = br/s
bis = bi/s
s = brs**2 + bis**2
cr = (ars*brs + ais*bis)/s
ci = (ais*brs - ars*bis)/s
return
end

subroutine csroot(xr,xi,yr,yi)
double precision xr,xi,yr,yi
C
C (yr,yi) = complex dsqrt(xr,xi)
C branch chosen so that yr .ge. 0.0 and sign(yi) .eq. sign(xi)
C
double precision s,tr,ti,pythag
tr = xr
ti = xi
s = dsqrt(0.5d0*(pythag(tr,ti) + dabs(tr)))
if (tr .ge. 0.0d0) yr = s
if (ti .lt. 0.0d0) s = -s
if (tr .le. 0.0d0) yi = s
if (tr .lt. 0.0d0) yr = 0.5d0*(ti/yi)
if (tr .gt. 0.0d0) yi = 0.5d0*(ti/yr)
return
end

subroutine corth(nm,n,low,igh,ar,ai,ortr,orti)
C
integer i,j,m,n,ii,jj,la,mp,nm,igh,kpl,low
double precision ar(nm,n),ai(nm,n),ortr(igh),orti(igh)
double precision f,g,h,fi,fr,scale,pythag
C
C this subroutine is a translation of a complex analogue of
C the algol procedure orthes, num. math. 12, 349-368(1968)
C by martin and wilkinson.
C handbook for auto. comp., vol.ii-linear algebra, 339-358(1971).
C
C given a complex general matrix, this subroutine
C reduces a submatrix situated in rows and columns
C low through igh to upper hessenberg form by
C unitary similarity transformations.
C
C on input
C
C nm must be set to the row dimension of two-dimensional
C array parameters as declared in the calling program
C dimension statement.
C
C n is the order of the matrix.
C

```

```

c      low and igh are integers determined by the balancing
c      subroutine cbal. if cbal has not been used,
c      set low=1, igh=n.
c
c      ar and ai contain the real and imaginary parts,
c      respectively, of the complex input matrix.
c
c      on output
c
c      ar and ai contain the real and imaginary parts,
c      respectively, of the hessenberg matrix. information
c      about the unitary transformations used in the reduction
c      is stored in the remaining triangles under the
c      hessenberg matrix.
c
c      ortr and orti contain further information about the
c      transformations. only elements low through igh are used.
c
c      calls pythag for dsqrt(a*a + b*b) .
c
c      questions and comments should be directed to burton s. garbow,
c      mathematics and computer science div, argonne national laboratory
c
c      this version dated august 1983.
c
c      -----
c
c      la = igh - 1
c      kpl = low + 1
c      if (la .lt. kpl) go to 200
c
c      do 180 m = kpl, la
c          h = 0.0d0
c          ortr(m) = 0.0d0
c          orti(m) = 0.0d0
c          scale = 0.0d0
c      ..... scale column (algol tol then not needed) .....
c      do 90 i = m, igh
90      scale = scale + dabs(ar(i,m-1)) + dabs(ai(i,m-1))
c
c          if (scale .eq. 0.0d0) go to 180
c          mp = m + igh
c      ..... for i=igh step -1 until m do -- .....
c      do 100 ii = m, igh
c          i = mp - ii
c          ortr(i) = ar(i,m-1) / scale
c          orti(i) = ai(i,m-1) / scale
c          h = h + ortr(i) * ortr(i) + orti(i) * orti(i)
100      continue
c
c          g = dsqrt(h)
c          f = pythag(ortr(m),orti(m))
c          if (f .eq. 0.0d0) go to 103
c          h = h + f * g
c          g = g / f
c          ortr(m) = (1.0d0 + g) * ortr(m)
c          orti(m) = (1.0d0 + g) * orti(m)
c          go to 105
c
103      ortr(m) = g

```

```

      ar(m,m-1) = scale
c ..... form (i-(u*ut)/h) * a .....
105 do 130 j = m, n
      fr = 0.0d0
      fi = 0.0d0
c ..... for i=igh step -1 until m do -- .....
      do 110 ii = m, igh
        i = mp - ii
        fr = fr + ortr(i) * ar(i,j) + orti(i) * ai(i,j)
        fi = fi + ortr(i) * ai(i,j) - orti(i) * ar(i,j)
110      continue
c
      fr = fr / h
      fi = fi / h
c
      do 120 i = m, igh
        ar(i,j) = ar(i,j) - fr * ortr(i) + fi * orti(i)
        ai(i,j) = ai(i,j) - fr * orti(i) - fi * ortr(i)
120      continue
c
130      continue
c ..... form (i-(u*ut)/h)*a*(i-(u*ut)/h) .....
      do 160 i = 1, igh
        fr = 0.0d0
        fi = 0.0d0
c ..... for j=igh step -1 until m do -- .....
        do 140 jj = m, igh
          j = mp - jj
          fr = fr + ortr(j) * ar(i,j) - orti(j) * ai(i,j)
          fi = fi + ortr(j) * ai(i,j) + orti(j) * ar(i,j)
140        continue
c
        fr = fr / h
        fi = fi / h
c
        do 150 j = m, igh
          ar(i,j) = ar(i,j) - fr * ortr(j) - fi * orti(j)
          ai(i,j) = ai(i,j) + fr * orti(j) - fi * ortr(j)
150        continue
c
160      continue
c
      ortr(m) = scale * ortr(m)
      orti(m) = scale * orti(m)
      ar(m,m-1) = -g * ar(m,m-1)
      ai(m,m-1) = -g * ai(m,m-1)
180      continue
c
200      return
      end

```

```

double precision function pythag(a,b)
double precision a,b

```

```

c
c finds dsqrt(a**2+b**2) without overflow or destructive underflow
c
double precision p,r,s,t,u
p = dmax1(dabs(a),dabs(b))
if (p .eq. 0.0d0) go to 20

```



```

      r = (dmin1(dabs(a),dabs(b))/p)**2
10 continue
      t = 4.0d0 + r
      if (t .eq. 4.0d0) go to 20
      s = r/t
      u = 1.0d0 + 2.0d0*s
      p = u*p
      r = (s/u)**2 * r
      go to 10
20 pythag = p
      return
      end

```

```

      subroutine cbal(nm,n,ar,ai,low,igh,scale)
c
c      integer i,j,k,l,m,n,jj,nm,igh,low,iexc
c      double precision ar(nm,n),ai(nm,n),scale(n)
c      double precision c,f,g,r,s,b2,radix
c      logical noconv
c
c      this subroutine is a translation of the algol procedure
c      cbalance, which is a complex version of balance,
c      num. math. 13, 293-304(1969) by parlett and reinsch.
c      handbook for auto. comp., vol.11-linear algebra, 315-326(1971).
c
c      this subroutine balances a complex matrix and isolates
c      eigenvalues whenever possible.
c
c      on input
c
c      nm must be set to the row dimension of two-dimensional
c      array parameters as declared in the calling program
c      dimension statement.
c
c      n is the order of the matrix.
c
c      ar and ai contain the real and imaginary parts,
c      respectively, of the complex matrix to be balanced.
c
c      on output
c
c      ar and ai contain the real and imaginary parts,
c      respectively, of the balanced matrix.
c
c      low and igh are two integers such that ar(i,j) and ai(i,j)
c      are equal to zero if
c      (1) i is greater than j and
c      (2) j=1,...,low-1 or i=igh+1,...,n.
c
c      scale contains information determining the
c      permutations and scaling factors used.
c
c      suppose that the principal submatrix in rows low through igh
c      has been balanced, that p(j) denotes the index interchanged
c      with j during the permutation step, and that the elements
c      of the diagonal matrix used are denoted by d(i,j). then
c      scale(j) = p(j),      for j = 1,...,low-1
c                = d(j,j)      j = low,...,igh
c                = p(j)      j = igh+1,...,n.

```

```

c   the order in which the interchanges are made is n to igh+1,
c   then 1 to low-1.
c
c   note that 1 is returned for igh if igh is zero formally.
c
c   the algol procedure exc contained in cbalance appears in
c   cbal in line. (note that the algol roles of identifiers
c   k,l have been reversed.)
c
c   arithmetic is real throughout.
c
c   questions and comments should be directed to burton s. garbow,
c   mathematics and computer science div, argonne national laboratory
c
c   this version dated august 1983.
c
c   -----
c
c   radix = 16.0d0
c
c   b2 = radix * radix
c   k = 1
c   l = n
c   go to 100
c   ..... in-line procedure for row and
c           column exchange .....
20  scale(m) = j
    if (j .eq. m) go to 50
c
c   do 30 i = 1, l
c       f = ar(i,j)
c       ar(i,j) = ar(i,m)
c       ar(i,m) = f
c       f = ai(i,j)
c       ai(i,j) = ai(i,m)
c       ai(i,m) = f
30  continue
c
c   do 40 i = k, n
c       f = ar(j,i)
c       ar(j,i) = ar(m,i)
c       ar(m,i) = f
c       f = ai(j,i)
c       ai(j,i) = ai(m,i)
c       ai(m,i) = f
40  continue
c
c   50 go to (80,130), iexc
c   ..... search for rows isolating an eigenvalue
c           and push them down .....
80  if (l .eq. 1) go to 280
    l = l - 1
c   ..... for j=l step -1 until 1 do -- .....
100 do 120 jj = 1, l
    j = l + 1 - jj
c
c   do 110 i = 1, l
c       if (i .eq. j) go to 110
c       if (ar(j,i) .ne. 0.0d0 .or. ai(j,i) .ne. 0.0d0) go to 120
110 continue

```

```

c      m = 1
        iexc = 1
        go to 20
120 continue
c
        go to 140
c      ..... search for columns isolating an eigenvalue
c      and push them left .....
130 k = k + 1
c
140 do 170 j = k, 1
c
        do 150 i = k, 1
            if (i .eq. j) go to 150
            if (ar(i,j) .ne. 0.0d0 .or. ai(i,j) .ne. 0.0d0) go to 170
150     continue
c
        m = k
        iexc = 2
        go to 20
170 continue
c      ..... now balance the submatrix in rows k to 1 .....
        do 180 i = k, 1
180     scale(i) = 1.0d0
c      ..... iterative loop for norm reduction .....
190 noconv = .false.
c
        do 270 i = k, 1
            c = 0.0d0
            r = 0.0d0
c
            do 200 j = k, 1
                if (j .eq. i) go to 200
                c = c + dabs(ar(j,i)) + dabs(ai(j,i))
                r = r + dabs(ar(i,j)) + dabs(ai(i,j))
200     continue
c      ..... guard against zero c or r due to underflow .....
            if (c .eq. 0.0d0 .or. r .eq. 0.0d0) go to 270
            g = r / radix
            f = 1.0d0
            s = c + r
210     if (c .ge. g) go to 220
            f = f * radix
            c = c * b2
            go to 210
220     g = r * radix
230     if (c .lt. g) go to 240
            f = f / radix
            c = c / b2
            go to 230
c      ..... now balance .....
240     if ((c + r) / f .ge. 0.95d0 * s) go to 270
            g = 1.0d0 / f
            scale(i) = scale(i) * f
            noconv = .true.
c
        do 250 j = k, n
            ar(i,j) = ar(i,j) * g
            ai(i,j) = ai(i,j) * g

```

```

      250      continue
c
      do 260 j = 1, l
        ar(j,i) = ar(j,i) * f
        ai(j,i) = ai(j,i) * f
      260      continue
c
      270 continue
c
      if (noconv) go to 190
c
      280 low = k
        igh = l
        return
      end

```

## APPENDIX B

### SYMBOLS

$\mathbf{a}$	=	Orthogonality matrix with respect to the $\mathbf{A}$ matrix
$\mathbf{b}$	=	Orthogonality matrix with respect to the $\mathbf{B}$ matrix
$\mathbf{A}$	=	$\mathbf{A}$ matrix
$\mathbf{B}$	=	$\mathbf{B}$ matrix
$c$	=	Damping of a single degree of freedom system
$\mathbf{C}$	=	Damping matrix of a $n$ by $n$ degree of freedom system
$\mathbf{F}$	=	Load vector of a $n$ by $n$ degree of freedom system
$\mathbf{F}(t)$	=	Forcing function vector of a $n$ by $n$ degree of freedom system
$k$	=	Stiffness of a single degree of freedom system
$\mathbf{K}$	=	Stiffness matrix of a $n$ by $n$ degree of freedom system
$m$	=	Mass of a single degree of freedom system
$\mathbf{M}$	=	Mass matrix of a $n$ by $n$ degree of freedom system
$\mathbf{N}$	=	Weighing matrix
$\mathbf{p}$	=	Inverse of Eigenvalue matrix, $m$ mode by $m$ modes
$\mathbf{q}$	=	Eigenvector, $n$ degree by $m$ modes
$\dot{\mathbf{q}}$	=	Eigenvelocity, $n$ degree by $m$ modes
$\ddot{\mathbf{q}}$	=	Eigenacceleration, $n$ degree by $m$ modes
$T$	=	Kinetic energy of the system
$U$	=	Strain energy of the system
$V$	=	Potential energy (due to strain and external forces) of the system
$W_{nc}$	=	Work done by nonconservative forces
$\delta$	=	Variation taken during two time intervals
$\lambda$	=	Eigenvalue matrix, $m$ mode by $m$ modes
$\ (\mathbf{matrix})\ ^2$	=	Weighted Euclidean norm of the matrix
$(\text{subscript}\mathbf{T})$	=	Refers to measured properties
$(\text{superscript}\mathbf{T})$	=	Transpose

## APPENDIX C

### LIST OF TABLES

Table	Page
I. Kinetic Energy Of The 12-BAR Truss	37
II. Test Damping Matrix Of The Four-Degree-Of-Freedom System	44
III. Eigenvalues Of Four-Degree-Of-Freedom System, 1-Mode Correlation, Method 1	45
IV. Eigenvelocities Of Four-Degree-Of-Freedom System, 1-Mode Correlation, Method 1	46
V. Eigenvalues Of Four-Degree-Of-Freedom System, 2-Mode Correlation, Method 1	47
VI. Eigenvelocities Of Four-Degree-Of-Freedom System, 2-Mode Correlation, Method 1	48
VII. Eigenvalues Of Four Degree Of Freedom System, 3-Mode Correlation, Method 1	49
VIII. Eigenvelocities Of Four Degree Of Freedom System, 3-Mode Correlation, Method 1	50
IX. Eigenvalues Of 12-BAR Truss, 2-Mode Correlation, Method 1, 95 Model	73
X. Eigenvelocities Of 12-BAR Truss, 2-Mode Correlation, Method 1, 95 Model	74
XI. Eigenvalues Of 12-BAR Truss, 3-Mode Correlation, Method 1, 95 Model	81
XII. Eigenvelocities Of 12-BAR Truss, 3-Mode Correlation, Method 1, 95 Model	82
XIII. Eigenvalues Of 12-BAR Truss, 4-Mode Correlation, Method 1, 95 Model	84
XIV. Eigenvelocities Of 12-BAR Truss, 4-Mode Correlation, Method 1, 95 Model	85

<b>Table</b>	<b>Page</b>
XV. Eigenvalues Of 12–BAR Truss, 5–Mode Correlation, Method 1, 95 Model	87
XVI. Eigenvelocities Of 12–BAR Truss, 5–Mode Correlation, Method 1, 95 Model	88
XVII. Eigenvalues Of 12–BAR Truss, 2–Mode Correlation, Method 2, 95 Model	93
XVIII. Eigenvelocities Of 12–BAR Truss, 2–Mode Correlation, Method 2, 95 Model	94
XIX. Eigenvalues Of 12–BAR Truss, 5–Mode Correlation, Method 2, 95 Model	95
XX. Eigenvelocities Of 12–BAR Truss, 5–Mode Correlation, Method 2, 95 Model	96
XXI. Eigenvalues Of 12–BAR Truss, 2–Mode Correlation, Method 1, 90 Model	102
XXII. Eigenvelocities Of 12–BAR Truss, 2–Mode Correlation, Method 1, 90 Model	103
XXIII. Eigenvalues Of 12–BAR Truss, 5–Mode Correlation, Method 1, 90 Model	104
XXIV. Eigenvelocities Of 12–BAR Truss, 5–Mode Correlation, Method 1, 90 Model	105
XXV. Eigenvalues Of 12–BAR Truss, 2–Mode Correlation, Method 2, 90 Model	109
XXVI. Eigenvelocities Of 12–BAR Truss, 2–Mode Correlation, Method 2, 90 Model	110
XXVII. Eigenvalues Of 12–BAR Truss, 5–Mode Correlation, Method 2, 90 Model	111
XXVIII. Eigenvelocities Of 12–BAR Truss, 5–Mode Correlation, Method 2, 90 Model	112

<b>Table</b>	<b>Page</b>
XXIX. Eigenvalues Of 44-BAR Truss, 2-Mode Correlation, Method 1, 90 Model	137
XXX. Eigenvelocities Of 44-BAR Truss, 2-Mode Correlation, Method 1, 90 Model	138
XXXI. Eigenvalues Of 44-BAR Truss, 5-Mode Correlation, Method 1, 90 Model	141
XXXII. Eigenvelocities Of 44-BAR Truss, 5-Mode Correlation, Method 1, 90 Model	142
XXXIII. Eigenvalues Of 44-BAR Truss, 2-Mode Correlation, Method 2, 90 Model	151
XXXIV. Eigenvelocities Of 44-BAR Truss, 2-Mode Correlation, Method 2, 90 Model	152
XXXV. Eigenvalues Of 44-BAR Truss, 5-Mode Correlation, Method 2, 90 Model	155
XXXVI. Eigenvelocities Of 44-BAR Truss, 5-Mode Correlation, Method 2, 90 Model	156



## APPENDIX D

### LIST OF FIGURES

Figure	Page
1. Kinetic Energy Values of 12-BAR Truss, 5 Mode Correlation, Energy of Mode 1 Is S1-1, Energy of Mode N Is SN-N	38
2. Four-Degree-Of-Freedom System	41
3. Analysis Finite Element Model Of 12-BAR Truss, Node Numbers	54
4. Analysis Finite Element Model Of 12-BAR Truss, Element Numbers	55
5. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 1, Frequency=162.5463 Hz, Phase=0	56
6. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 2, Frequency=609.2841 Hz, Phase=0	57
7. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 3, Frequency=945.7958 Hz, Phase=0	58
8. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 4, Frequency=1596.589 Hz, Phase=0	59
9. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 1, Frequency=162.5463 Hz, Phase=-90	60
10. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 2, Frequency=609.2841 Hz, Phase=-90	61
11. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 3, Frequency=945.7958 Hz, Phase=-90	62
12. Modal Deformation Plot Of Analysis Finite Element Model Of 12-BAR Truss, Mode 4, Frequency=1596.589 Hz, Phase=-90	63
13. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 1, Frequency=162.6529 Hz, Phase=0	64
14. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 2, Frequency=604.2818 Hz, Phase=0	65

<b>Figure</b>	<b>Page</b>
15. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 3, Frequency=940.3127 Hz, Phase=0	66
16. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 4, Frequency=1581.193 Hz, Phase=0	67
17. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 1, Frequency=162.6529 Hz, Phase=-90	68
18. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 2, Frequency=604.2818 Hz, Phase=-90	69
19. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 3, Frequency=940.3127 Hz, Phase=-90	70
20. Modal Deformation Plot Of Test Finite Element Model Of 12-BAR Truss, 95 Model, Mode 4, Frequency=1581.193 Hz, Phase=-90	71
21. Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.11), Analysis Model, 2 Mode System Identification	75
22. Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.11), Test Model, 95 Model, 2 Mode System Identification	76
23. Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10), Test Modal Properties, and Analytical Spatial Properties, 2 Mode System Identification Using Method 1	77
24. Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10), Test Modal Properties, and Test Spatial Properties, 2 Mode System Identification Using Method 1	78
25. Orthogonality Condition Of 12-BAR Truss Based On Eqn. (3.10), Correlated Modal Properties, and Test Spatial Properties, 2 Mode System Identification Using Method 1	79

<b>Figure</b>	<b>Page</b>
26. Orthogonality Condition Of 12–BAR Truss Based On Eqn. (3.10), Correlated Modal Properties, and Test Spatial Properties, 5 Mode System Identification Of 95 Model Using Method 1	91
27. Orthogonality Condition Of 12–BAR Truss Based On Eqn. (3.10), Correlated Modal Properties, and Test Spatial Properties, 5 Mode System Identification Of 95 Model Using Method 2	99
28. Orthogonality Condition Of 12–BAR Truss Based On Eqn. (3.11), Correlated Modal Properties, and Test Spatial Properties, 5 Mode System Identification Of 95 Model Using Method 2	100
29. Analysis Finite Element Model Of 44–BAR Truss, Node Numbers	118
30. Analysis Finite Element Model Of 44–BAR Truss, Element Numbers	119
31. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 1, Frequency=19.09284 Hz, Phase=0	120
32. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 2, Frequency=22.44332 Hz, Phase=0	121
33. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 3, Frequency=33.76233 Hz, Phase=0	122
34. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 4, Frequency=64.01659 Hz, Phase=0	123
35. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 1, Frequency=19.09284 Hz, Phase=–90	124
36. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 2, Frequency=22.44332 Hz, Phase=–90	125
37. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 3, Frequency=33.76233 Hz, Phase=–90	126
38. Modal Deformation Plot Of Analysis Finite Element Model Of 44–BAR Truss, Mode 4, Frequency=64.01659 Hz, Phase=–90	127
39. Modal Deformation Plot Of Test Finite Element Model Of 44–BAR Truss, 90 Model, Mode 1, Frequency=18.95565 Hz, Phase=0	128

<b>Figure</b>	<b>Page</b>
40. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 2, Frequency=22.21710 Hz, Phase=0	129
41. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 3, Frequency=32.91091 Hz, Phase=0	130
42. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 4, Frequency=62.99915 Hz, Phase=0	131
43. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 1, Frequency=18.95565 Hz, Phase=-90	132
44. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 2, Frequency=22.21710 Hz, Phase=-90	133
45. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 3, Frequency=32.91091 Hz, Phase=-90	134
46. Modal Deformation Plot Of Test Finite Element Model Of 44-BAR Truss, 90 Model, Mode 4, Frequency=62.99915 Hz, Phase=-90	135

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE May 1995		3. REPORT TYPE AND DATES COVERED Technical Memorandum	
4. TITLE AND SUBTITLE  System Identification of Damped Truss-Like Space Structures				5. FUNDING NUMBERS  WU-505-69-50	
6. AUTHOR(S)  Sasan Armand					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191				8. PERFORMING ORGANIZATION REPORT NUMBER  E-8941	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  National Aeronautics and Space Administration Washington, D.C. 20546-0001				10. SPONSORING/MONITORING AGENCY REPORT NUMBER  NASA TM-106638	
11. SUPPLEMENTARY NOTES This report was submitted as a dissertation in partial fulfillment of the requirements for the degree Doctor of Philosophy in Mechanical Engineering to Cleveland State University, Cleveland, Ohio, March 1994. Responsible person, Sasan Armand, organization code 2410, (216) 433-7040.					
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Unclassified - Unlimited Subject Category 37  This publication is available from the NASA Center for Aerospace Information, (301) 621-0390.				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  A spacecraft payload flown on a launch vehicle experiences dynamic loads. The dynamic loads are caused by various phenomena ranging from the start-up of the launch vehicle engine to wind gusts. A spacecraft payload should be designed to meet launch vehicle dynamic loads. One of the major steps taken towards determining the dynamic loads is to correlate the finite element model of the spacecraft with the test results of a modal survey test. A test-verified finite element model of the spacecraft should possess the same spatial properties (stiffness, mass, and damping) and modal properties (frequencies and mode shapes) as the test hardware representing the spacecraft. The test-verified and correlated finite element model of the spacecraft is then coupled with the finite element model of the launch vehicle for analysis of loads and stress. Modal survey testing, verification of a finite element model, and modification of the finite element model to match the modal survey test results can easily be accomplished if the spacecraft structure is simple. However, this is rarely the case. A simple structure here is defined as a structure where the influence of nonlinearity between force and displacement (uncertainty in a test, for example, with errors in input and output), and the influence of damping (structural, coulomb, and viscous) are not pronounced. The objective of this study is to develop system identification and correlation methods with the focus on the structural systems that possess nonproportional damping. Two approaches to correct the nonproportional damping matrix of a truss structure were studied, and have been implemented on truss-like structures such as the National Aeronautics and Space Administration's space station truss. The results of this study showed nearly 100 percent improvement of the correlated eigensystem over the analytical eigensystem. The first method showed excellent results with up to three modes used in the system identification process. The second method could handle more modes, but required more computer usage time, and the results were less accurate than those of the first method.					
14. SUBJECT TERMS  Structural vibration; Vibration tests				15. NUMBER OF PAGES 281	
				16. PRICE CODE A13	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT		